A MULTIVARIATE STOCHASTIC TIME SERIES MODEL FOR BLAST FURNACE

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By
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AT

THE FEET OF

MY

REVERED PARENTS

CERTIFICATE

This is to certify that the thesis entitled ' A MULTIVARIATE STOCHASTIC TIME SERIES MODEL FOR BLAST FURNACE' submitted by Mr. S.K.Saxena, in partial fulfilment of the requirements for the Degree of Master of Technology at the Indian Institute of Technology, Kanpur is a record of bonafide research work carried out under our supervison. The work embodied in this thesis has not been submitted elsewhere for a degree.

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Author

CONTENTS

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ABSTRACT

In the present study mathematical models for the blast furnace process have been reviewed. A decoupled multivariate stochastic model has been developed using multivariate time series analysis. This method is a generalization of the method proposed earlier by Phadke et al.

Data have been collected for the blast furnace at Bokaro Steel Limited, Bokaro Steel City. The input variables considered in the analysis were sinter-to-coke ratio, blast humidity and blast flow rate. The output variables taken into account were hot metal temperature, silicon and sulphur contents of hot metal. The original series were first 'prewhitened' by means of unvariate modelling. The prewhitened series which are normally distributed and serially independent were used in multivariate time series analysis.

The method uses Gram-Schmidt procedure to obtain orthogonal vectors. A principal component model was derived in terms of prewhitened series and orthogonal vectors. A multivariate model was then developed using cross-correlation function between orthogonal vectors and multiple regression analysis. The orthogonal vectors were found to be serially and mutually uncorrelated andhence the multivariate model was also a principal component model.

Using this multivariate model, transfer function models were developed, both in terms of the prewhitened input-output variables; and in terms of actual input-output variables by recoupling the multivariate model. It was found that the input-output relationship consists of terms involving shift operator which represent the time delay and other terms which represent the time constant. The average time lag between each input and output variables was found to be of the order of 0.85 to 1.10 cast intervals.

These results are generally in agreement with earlier results. But they indicate greater details of the transformation relationships. Particularly important are the direct interactions of the order of 2 to 3 cast intervals between the input and output variables. However no feedback control of the order of one cast interval or more were indicated in the study.

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LIST OF TABLES

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LIST OF FIGURES

+++++++

LIST OF SYMBOLS

$\underline{\underline{A}}_i$	Autoregressive coefficient matrix (Eqn. 4.1)
AR	Autoregressive
ARIMA	Autoregressive - integrated - moving average
ARMA	Autoregressive-moving average
$a_1, a_2 \ldots$	Constants
B	Backward shift operator
$\underline{\underline{B}}_i$	Moving average coefficient matrix (Eqn.4.1)
b_{ij}	Dead time between ith input and jth output variable
$b_1, b_2, \ldots$	Constants
$\underline{\underline{C}}(B)$	m x n matrix of feedback dynamics
$\underline{\underline{C}}_i$	Autoregressive coefficient matrix in decoupled multivariate model
CL	Confidence limit
c_i'	Amplitude of the ith harmonic
cov	Covariance
$c_o, c_1, \ldots$	Constants
D	Differential operator
$\underline{\underline{D}}_i$	Moving average coefficient matrix in decoupled multivariate model
d	Degree of differencing
E	Expectation operator
$E_i(t)$	Standardized univariate residual series at time t
e(t)	Residual series at time t obtained from backward model (Eqn. 3.46)
F	Forward shift operator; cumulative distribution function

f_c	Cut-off frequency
f_i	Frequency of ith harmonic
$f(i)$	Observed frequency of the sample from ith class
f_j'	Error vector (Eqn. 3.42)
$\tilde{G}(f), \tilde{G}_i$	Spectral density function at the frequency f
$G(f), G_i$	Smooth spectral density function at the frequency f
$\underline{\underline{H}}$	Lower triangular matrix for principal component model (Eqn. 4.33)
I	Number of classes in frequency analysis
J	Number of parameters in a distribution
j	Square root of -1
K	Total number of input-output variables considered.
k	Lag in casts
L(B)	Transfer function matrix in terms of pre-whitened input-output variables
L'(B)	Transfer function matrix in terms of actual input-output variables
$L(\xi'/z)$	Likelihood function
$l(\xi'/z)$	Log likelihood function
M	Maximum number of lags upto which autocorrelation function is computed
MA	Moving average
$\underline{\underline{M}}_o$	Lagzero correlation matrix
$\underline{\underline{M}}_{-1}$	Lagone correlation matrix
$M_i(t)$	Feedback disturbance
m	Number of input variables

N	Sample size (Total number of data points)
$N_i(t)$	Plant disturbance
n	Number of output variables
p	Order of autoregression
$p(i)$	Probability that a variable belongs to ith class
$p(z/\xi')$	Probability distribution function
Q	Q-statistic (Eqn.3.59)
q	Order of moving average
R_k	Estimated autocovariance function at lag k
r_k	Estimated autocorrelation function at lag k
$r_{xy}(k)$	Estimated cross-correlation function between series x and y at lag k
SE	Standard error
$S(\underline{\varphi},\underline{\theta}), S(\underline{\xi})$	Sum of squared errors in ARMA model
T	Total time
T_c	Total time constant
$T_{c_{\bar{2}}}, T_{c_2}$	Time constants
T_D	Dead time (time delay)
t	Time
$\underline{\underline{U}}(B)$	Diagonal matrix representing univariate relationships
$\underline{u}(t)$	Vector of independent random components (Eqn. 4.53)
$\underline{\underline{V}}(B)$	n x m matrix representing blast furnace dynamics
$\underline{\underline{V}}'(\underline{\xi})$	Variance covariance matrix of parameters in ARMA model
var	Variance
$v_1, v_2, ..$	Constants

W	Total weightage on an input
W_1 to W_5	Weightages for the operators B^2, B, 1, T_{c_1} and T_{c_2}, respectively
w_i	Angular frequency
$w(t)$	Stationary differenced series
$\underline{X}(t)$	Vector of m-input variables
$x(t)$	Value of the cycle free series at time t
$\underline{x}(t)$	Vector of all (n+m) variables
$\bar{x}(t)$	Deviation from meanof x(t)
$\underline{Y}(t)$	Vector of n-output variables
$y(t)$	Standardized series (mean zero and unit variance) at time t
$\underline{Z}(t)$	Vector of input and output variables
$z(t)$	Value of the raw data at time t
z	z-transform

Greek Letters

$\alpha_i(t)$	Value of ith orthonormal series at time t
α	Significance level
$\beta_i(t)$	Value of ith orthogonal series at time t
γ_k	Theoretical autocovariance function at lag k
$\gamma_{xy}(k)$	Theoretical cross-covariance function between series x and y at lag k
∇	Operator for differencing
Δt	Time interval between successive values of the discrete data
δ_{ij}	Dot product between $\alpha_i(t)$ and $\varepsilon_j(t)$

$\varepsilon_j(t)$	Value of jth univariate residual series at time t
$\xi'_i(t)$	ith whitenoise series, (Eqn. 4.39)
$\xi''_j(t)$	jth whitenoise series (Eqn. 4.41)
θ_i	ith moving average parameter
θ'_i	Phase angle of ith harmonic
μ	Population mean
ν	Number of degrees of freedom (Eqn. 3.60)
$\underline{\xi}$	Vector of parameters, (p+q) of ARMA model
$\underline{\xi}'$	Vector of total parameters (p+q+2)
ρ_k	Theoretical autocorrelation function at lag k
$\rho_{xy}(k)$	Theoretical cross-correlation function between series x and y at lag k
Σ_u	Variance-covariance matrix of the vector $\underline{u}(t)$
σ_x	Population standard deviation of x
$\underline{\tau}$	Standardized MA parameter vector of ARMA model (Eqn. 3.41)
φ_i	ith autoregressive parameters
φ_{io}	Initial estimate of ith autoregressive parameter
φ_{kk}	Theoretical partial autocorrelation function at lag k
χ^2	Chisquare statistic
$\underline{\underline{\Psi}}(B)$	Lower triangular matrix of multivariate model, (Eqn. 4.5)
$\underline{\underline{\Psi}}'(B)$	Matrix representing blast furnace noise
$\underline{\underline{\Psi}}''(B)$	Matrix representing feedback noise

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CHAPTER 1

INTRODUCTION

1.1 GENERAL:

Blast furnace is a major equipment of an integrated iron and steel plant which produces pig iron or hot metal of desired quality. The principal raw materials used in the blast furnace are iron ore, sinter, coke, limestone and manganese ore. Other materials such as quartzite, dolomite, scrap, open hearth slag etc are used depending upon the requirement and their availability.

Fundamentally, the blast furnace is a counter-current apparatus in which chemical and heat transfers take place. The solids added at the top of the furnace are usually coke and sinter. Other materials are added to take care of fluctuations in the sinter composition and/or to get the desired composition of the metal. Sinter is an agglomerate of ore fines, limestone and some essential additives which facilitate the production of iron. The gaseous phase is a mixture of CO, CO_2, H_2, H_2O and N_2 which is more and more oxidized as it ascends the furnace and releases heat. The initial gas constituents are CO, H_2 and N_2 at high temperature. They result from combustion of a part of coke by the blast air and decomposition of steam injected along with the blast. The blast air is preheated to about $1000^{\circ}C$. The gas heats the

solid burden, reduces iron oxides to iron which is finally melted and collected in the hearth. The slag which is formed with the gangue material floats on molten pig iron. These liquid products are tapped after every two to three hours.

The blast furnace is characterized by a considerable number of inputs and outputs which together define the operating status of the furnace. The inputs or independent variables are defined as those which affect the behaviour of the system. These variables include those which can be directly manipulated by the operator (manipulatable variables or controllable variables) as well as those which are beyond his direct control (disturbances or noise in the system). Outputs or dependent variables are the remaining variables of the process which are the result of the input variables and are needed to describe the operating conditions of the process. In other words these are responses of the system to the inputs. The manipulatable variables may be classified into two principal categories: charge variables such as sinter-to-coke ratio; and operating variables such as blast temperature, blast humidity and blast flow rate. The smooth operation of the blast furnace is affected by many disturbing factors. The significant disturbances are chemical and physical composition of the charge material, porosity of the bed etc. The outputs or dependent variables are the temperature of hot metal and its sulphur and silicon content.

1.2 NEED FOR A MODEL:

The advent of fast and sophisticated L.D. steelmaking process demands great improvements in the control of iron-making process and in particular better control of silicon and sulphur content of the molten metal. Models of the furnace dynamics are needed to design a control scheme to minimize variations in the silicon content in pig iron from cast to cast. A well regulated silicon content is likely to lead to smooth and economical ironmaking as well as steelmaking processes. The consistent silicon content implies stable thermal conditions in the lower part of the furnace. Studies of control systems are based on a mathematical model of the process involved. A large number of blast furnace models have been proposed in past ranging from completely theoretical to fully empirical ones. A review has been published earlier [1].

Blast furnace contains a number of inputs and outputs and in order to achieve optimal control, it is necessary to identify (i) the transfer function relating each manipulatable variable with each output and (ii) the disturbance or noise present in the system. The classical methods of estimating the transfer functions are based on studying the result of specific deterministic perturbations of an input while maintaining other inputs at their steady levels. The perturbation may be of the form of a step, pulse or sinusoid. However, such procedures have not always been successful. This is because

for perturbations of a magnitude that are relevant and tolerable, the response of the system may be masked by uncontrollable disturbances in the system, and the large capacity of the system may absorb permissible magnitudes of the perturbation. Further other inputs have to be maintained at their steady levels which is very difficult in the plant environment.

The sequence of values of a variable observed along time is called a time series. The sequencesof values of one variable constitutes a univariate time series. The sequences of values of number of variables constitute a multivariate time series. The statistical analysis of multivariate time series takes into account the disturbances present in the system. Multivariate modelling involves the representation of the relationships among input and output variables and is considered in this study.

1.3 OBJECTIVES OF THE STUDY:

The objectives of the present study are as follows:

(a) to fit a suitable univariate model to each of the variables under consideration so as to obtain residual series which are pure random. If the residuals have a normal distribution, they may be called as 'prewhitened series'.

(b) to fit a suitable multivariate model to the serially independent residual series in terms of serially and mutually independent random components, and

(c) to validate the fitted multivariate model and hence derive by suitable matrix operations, a transfer function model from the multivariate model.

This multivariate model is called a decoupled model because, all series are first prewhitened by univariate time series analysis. This approach for modelling multiple time series is advantageous because:

(i) as the parameters of univariate and multivariate models are estimated separately, their numbers are comparable to those of univariate and multivariate models and the difficulties involved in simultaneously estimating all parameters are avoided; and

(ii) the most appropriate form and order of model can be chosen independently for each variable, and hence there is greater flexibility in the choice of the models.

1.4 ORGANIZATION OF THE STUDY:

The study is reported in the following sequence:

(i) Chapter 2 is a review of mathematical models that have been proposed in the past.

(ii) The technique of univariate time series analysis are described in Chapter 3 in the following sequence - First the general time series models are described and then the similarity between the linear difference equation and linear differential equation is given. The steps for fitting univariate time series models are then described. These include:

identification of a suitable model, estimation of the parameters of the identified model and the diagnostic checking of the residuals for their normality and serial independence. They are applied to field data from Bokaro Steel Plant.

(iii) In Chapter 4, a multivariate model is fitted to the serially uncorrelated residuals of the unvariate models. The method proposed by Phadke et al. [2] has been generalized and used in the modelling of the process. The transfer function model is then developed from the decoupled multivariate time series model.

(iv) Finally, in Chapter 5 conclusions are drawn on the basis of results of the present study and some suggestions for future work are given.

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CHAPTER 2

MATHEMATICAL MODELS OF BLAST FURNACE PROCESS

The mathematical models for blast furnace process may broadly be classified as deterministic or probabilistic models. A general classification of the models is given in Table 2.1.

2.1 DETERMINISTIC MODELS:

A deterministic model is one where the future behaviour of a physical system is expressed uniquely by a set of algebraic, differential or integro-differential equations and hence there is no uncertainty or randomness in the future outcomes. Deterministic models may further be subdivided into two groups- steady-state and dynamic models.

2.1.1. Steady-State Models:

Steady state models deal with equilibrium conditions of a process. Considerable amount of work has been done in past on formulation of mathematical models for blast furnace assuming steady state conditions, i.e. assuming that the operating conditions of the furnace do not change with time. These models are based on mass and energy balances and may be divided into three groups:

1. Models based on overall heat- and material balances
2. Models based on stagewise heat- and material balances

3. Models based on differential heat and material balances

Models Based on Overall Heat and Material Balances: Overall balances are commonly used to check the consistency of the plant data. Models belonging to this group include those of Joseph et al.[3], Marshall [4], Dancy et al. [5], Lander et al. [6-7], Fazzalari et al [8], Khromov et al [9], Lahiri et al.[10], and Pokhvisnev et al [11-13] and have been summarized in Table 2.2.

Joseph et al [3] have derived carbon balance, oxygen balance and heat balance from the known operating conditions and the average top gas analysis. Marshall [4], related blast furnace operating variables with coke rate and production rate with the help of carbon balance, heat balance, wind rate and coke burnt at the tuyeres. Dancy et al [5] calculated the production rate and coke rate for a blast furnace with oxygen enrichment and steam injection through enthalpy and oxygen balances. It was assumed that the amount of heat required to produce a unit of iron from a given burden is constant irrespective of blast composition, provided that the burden composition, slag composition and the stone ratio do not alter, though the reactivity of the raw material, the burden size, porosity, flow pattern all have effect on the amount of energy required to reduce the ore to iron. The model was applicable to changes

WORKER	AIM OR CONTROL OBJECTIVE	EQUATI
1	2	3
Joseph. et. al. (1947)	Coke rate and production rate	Calculation proceedure is descrit given operating conditions.
Marshall (1947)	Coke rate and production rate	Equations for carbon balance, hea relating production, wind rate tuyeres.
Dancy et. al. (1958)	Coke rate and production rate	Oxygen balance: $N'(2X'_{O_2} + X'_{H_2O}) +$ $+X'_{H_2O}) + d] +$ Heat Balance : $\Delta H_{Fe} = N'[X'_{O_2} H'_{O_2} + X'_{N_2} H'_{N_2}$ $- \frac{X''_{CO_2}}{X''_{CO} + X''_{CO_2}} \frac{N'(2X'_{O_2} + X'_{H_2O}) + d] H''_{C}}{[N^{c} + N'(2X'_{O_2} +}$ Production : $P^* = \frac{P_o N' X'_{N_2}}{N'^* \cdot X'^*_{N_2}}$ Coke Rate = $\frac{\text{Nc R x (lb atom Fe/}}{\text{(\% C in Coke) x (TH}}$
Lander et. al (1960)	Coke rate and production rate	Carbon Balance: $N'(2X'_{O_2} + X'_{H_2O})$ Oxygen balance: $N_o^B + N_o^L + N'(2X'$ $[N_c^L + N'(2X'_{O_2}$ Heat balance : $[H^{Hm} + M^S - H^B - $ $= N'[X'_{O_2} H'_{O_2} +$ $- H''_{N_2}(N' X'_{N}$ Productivity : $p = \frac{N'^* X'^*_{1}}{N' X'}$

S	DRAW BACKS
for a blast furnace	conditions.
balance and an equation carbon buried at the	The equations did not include all thermal and Chemical variables of the process.
$+N_O^B = \frac{X''_{CO}+2X''_{CO_2}}{X''_{CO}+X''_{CO_2}}\left[N_C^L + N'(2X'_{O_2}\right.$ ... $_{H_2O}\left[\frac{X''_{H_2O}}{X''_{H_2}+X''_{H_2O}}\right]$ ... $_{H_2O} \cdot H'_{H_2O}] - \frac{X''_{CO}}{X''_{CO}+X''_{CO_2}}[N^L +$ ... $N'X_{N_2}H''_{N_2} - N'X'_{N_2}K$ ... $_{2O}) + d] H''_{CO_2} - \frac{X''_{H_2}}{X''_{H_2}+X''_{H_2O}}[N'X'_{H_2O}]H''_{H_2O}$ ... in reference period) ... ay in reference period)	It was assumed that the amount of heat required to produce a unit of iron is constant, Which is determined from reference period of operation. This assumption introduces serious errors. The system of equations is applicable only to changes in performance brought about by blast modification.
$\ldots = N_C^{C*} + \Delta N_C + N_C^B + N_C^{FD} - N_C'^{HM}$ ... $\cdot X'_{H_2O}) = \left[\frac{2+R_C}{1+R_C}\right]$ ... $_{H_2O}) + d] + \left[\frac{1}{1+R_H}\right] N'X'_{H_2O}$ ... $^L] = [H' - H'' - H_{heat\ loss}]$	Though the model can be used to predict the effect of zone of the operating variables on blast furnace performance and for production planning, optimizing the burdens with regard to cost and productivity, it cannot predict what changes in blast temp. and wind rate is compatible with smooth operation of the furnace if major burden changes is made.

$$M'_{N_2} + X'_{H_2O} M'_{H_2O}] - \frac{(R_C H''_{CO} + H''_{CO_2})}{1+R_C}\left[N_C^L + N'(2X'_{O_2} + X'_{H_2O}) + d\right]$$

$$- K N' X'_{N_2} - \frac{R_H M''_{H_2} + H''_{H_2O}}{1+R_H} N' X'_{H_2O}$$

$$P^* \; ; \; \text{Coke Rate} = 2000\left[\frac{\%Fe^{HM}}{MW_{Fe}}\right]\left[\frac{MW_C}{\%C_C}\right] * \left[N_C^{C*} + \Delta N_C\right]$$

Contd...2/-

1	2	3
Fazzalari et. al. (1963)	Production rate, coke rate, shaft gas efficiency, combustion zone temp., absolute top gas temp.	Carbon balance: $C_{DRI} = C_{gasified} - (C_{tuyeres} + C_{metal\ldots}$ $C_{gasified} = C_{charged} - C_{HM} - C_{dust.\ldots}$ H_2 Balance: $H_2O_{blast} + \frac{H_{nat.gas.}}{0.1119} + \frac{M_{coke}}{0.1119} + \ldots$ $H_2O_{raw\ mat.} = \frac{H_{top\ gas}}{0.1119} + H_2O_{top\ ga\ldots}$ Shaft efficienty = $\frac{\text{Fe reduced by C}\ldots}{\text{Total Fe reduce}\ldots}$ Production = $\frac{C_{top\ gas} - C_{nat.gas}}{C_{burden} + C_{stove} + C_{c\ldots}}$ Rate of coke input: Coke = $\frac{C_{top\ gas} + C_{HM} - C_{burden} - \ldots}{\text{weight fraction of carbo}\ldots}$ Instantaneous coke rate =
Khromov (1969)	Control of thermal state	The heat unbalance - ΔQ $\Delta Q = \frac{Q_{reg} - Q_{cal}}{Q_{ram}} \times 100.$ Qs…
Lahiri (1969)	Coke rate and optimum gasification	Carbon balance: Total carbon from … supply + Carbon for gasification + … lost as dust. Enthalpy balance : $\frac{26420}{12} \times C_c = H\ldots$ $+ H_{hot\ metal} + H_{slag} + H_{top\ gas} + \ldots$

Table-II (Contd.)

	4
calloids + $C_{carbonates}$) ust. + H_2O ore gas + H_2O_{dust}. r CO ued. $C_{coke} - C_{hot\ metal}$ Cstove − $C_{nat.gas}$ / bon in coke Coke / Production	Combustion zone temp. and absolute top gas temp. are calculated by carrying out heat balance of the lower region and upper region respectively.
$\frac{Qsul - Qcal}{Qcal}$	Qcal is calculated by heat and material balances. The controllable variable is Ore - to - coke ratio. Qsul - is actual supplied
n coke = Carbon for heat + Carbon in soln + Carbon $H_{reactions}$ + $H_{heat\ loss}$ − H_{blast}	The influence of charge composition and other paramters on optimum gasification and coke rate have been analyzed. Some of the results obtained were not in agreement with actual observations.

brought about by blast modification only. Lander et al [6,7] modified Dancy's approach which enabled them to calculate the effect of variations in the burden composition on the production rate and the coke rate. This was accomplished by defining a quantity known as furnace characteristic constant obtained from reference operating conditions. The composition and temperature of top gas were assumed to be similar to that of reference conditions. The model was applicable only to small perturbations in the operating conditions from that of the reference conditions. Fazzalari et al. [8] developed a computer model which consisted of overall carbon balance, heat balance, absolute top gas temperature, theoretical flame temperature,production rate, coke rate and shaft efficiency. Khromov et al. [9] developed a model for stabilization of blast furnace based on a variable defined as heat unbalance , that is caused by previous charging cycles. The heat unbalance was calculated as the difference between the heat required and the heat available in the furnace per kg of the carbon in the dry top gas divided by the heat available. Lahiri et al. [10] developed a model for calculating coke rate and optimum gasification rate utilizing the principle of optimum gasification. He assumed that carbon monoxide is mainly supplied by the exothermic reaction $C + \frac{1}{2}O_2 = CO$ and only the deficient amount is supplied by the endothermic gasification reaction $C + CO_2 = 2CO$. The influence of charge composition and other parameters on coke rate were studied. The effects of

gangue in ore, of ash in the coke and of silica in the flux on the coke rate as predicted by the model were not as significant as in the actual practice. Pokhvisnev et al.[11-13] developed two thermal state indices M^g and M^{ch} for monitoring thermal state of the blast furnace. Indices M^g and M^{ch} represented the heat input in the blast furnace per unit of the oxygen of the charge gasified. M^g was calculated from the top gas composition and M^{ch} was calculated from the composition of the charge, blast and injected additives. Both indices had positive correlation with silica content of pig iron.

Models Based on Stagewise Heat and Material Balances: Models which belong to this group were presented by Reichardt [14], Ridgion [15], Hodge [16-18], Pokhvisnev et al.[19], Dovgalyuk et al. [20], Wartmann [21], Staib et al.[22-24], Daconsine[25], VanLangen et al.[26-30], Flierman et al. [31] and Rist et al.[32] and have been summarised in Table 2.3.

Reichardt [14] studied the longitudinal distribution of temperature of gas and solid particles in the furnace and demonstrated the existence of the 'thermal pinch point' which is associated with the sudden increase in the overall capacity of the charge. The thermal pinch point is defined as a point where the solids and gaseous phases have the same temperature. Ridgion [15] developed a model for stagewise heat balance assuming that the temperature of the gas at any level in the

	AIM OR CONTROL OBJECTIVE	NO. NO. OF REGIONS	CONTROLABLE
	2	2	4
	Stagewise heat balance (heat required for solid and gaseous phases).	Heat requirement of solid phase is calculated in 50°C steps from 0°C to 1500°C. Reactions occurring are divided in 3 groups:- Relating to burden constituents other than iron compounds; iron reduction, and soln loss.	-
	Coke rate and Prodction rate	Three:- (1) Removal of moisture (2) Preheating, calcination, indirect reduction, direct reduction, reduction with H_2 (3) Direct reduction, slag formation, liquification and solution of elements, high temp. combustion.	
t.al.	Automatic control of the thermal state of blast furnace.	Two:- Direct reduction (Lower Zone), and indirect reduction (Upper Zone).	For upper size of co or propert natural g lower zone temp., bla humidity.
t.al.	Computer control of the thermal state of the blast furnace	Two:- Direct reduction and indirect reduction zones.	Ore to - Coke rati

$\Delta Mo =$

VARIABLE	EQUATIONS	DRAWBACKS
	45	6

A computer programme is written including all input and output terms and results presented as graph between heat and temperature.

The assumption that the temp. of gas at any level is higher than that of corresponding solid is no in agreement with actual observation.

Heat balance:

$$\left[S_b(T-T_1)+(G_d-G_b')(t_x-T_1)+J\right]=\frac{K\theta^{n}\psi}{2\cdot3G_b}\left[\log\frac{(G_b-G_d)(t_x-T_1)+(G_b-S_b)(T-T_1)-J}{G_b(t_x-T_1)+(G_b-S_b)(T_1-T)+J}\right]$$

Carbon for direct reduction:

$$C=(c-5\cdot96eH)+a\log\left[\frac{\theta r d_1 d_2(d_1'+r'd_2')}{(d_1+rd_2)(r'd_1+d_2')}\right]$$

Relative shaft gas velocity:

$$RGV=\frac{\theta V'(t'_x+460)P}{V(t_x+460)P'}$$

The assumption that the quantity of carbon consu by direct reduction depen on residence time of soli particles-residence time solid presents gross oversimplification.

The model cannot yield g results for appreciable iation from the standard case.

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one –
e charge/
onal of
. For
blast
t

Heat input in lower zone:

$$M_l=\frac{1245(CO+CO_2)+\alpha N_2 W_{cb}}{O_{ch}}$$

Heat input in upper zone:

$$M_u=\frac{3021\,CO_2+\Sigma H_2O\times2580}{O_{ch}}$$

The problem of thermal control is to bring the present valves of M_4 and M close to the normal valves of heat inputs a which pig iron of requir composition is obtained.

$$\left\{(12\cdot65\,CO_2+10\cdot8\,\Delta H_2)\frac{V_{cb}}{\alpha N_2}+\left[5\cdot25(CO+CO_2)+\alpha N_2 W_{cb}\right]\times\frac{V_{cb}}{\alpha N_2}-0\cdot0685\frac{L}{t_c}\right\}$$

$$0\cdot5(CO_2+\Delta H_2)\frac{V_{cb}}{\alpha N_2}+0\cdot5(CO+CO_2)-\beta N_2\Big]\frac{V_{cb}}{\alpha N_2}-0\cdot00356\frac{L}{t}\Big\}$$

is calculated as dif erence between the heat input unto the furnace a a whole and the heat inpu into the lower part of t furnace. Computer calcul tes and compares it with the normal value ar then recommends the char

$$\Big\{(12\cdot65\,CO_2^7+10\cdot8\,\Delta H_2^7)\Big(\frac{V_{cb}}{\alpha N_2}\Big)^7+\big[5\cdot25(CO+CO_2)\ldots\big]\frac{V_{cb}}{\alpha N_2}-0\cdot068\frac{L}{t}\Big\{$$

	2	3	4	
	Control of the thermal state of the blast furnace	-	Ore-to-coke ratio $M^g =$	$M_{ch} =$
	Predicting blast furnace preformance under various changes in operating variables.	Two: Preparation zone in which burden is preheated and reduced to constite, Elaboration zone in which direct reduction, soln loss and combustion of coke take place.	-	oxygen $x_1 + 2$ Therm $\alpha_1 (46$ $+ 46.12$ $17x_1 +$
.)	Stabilization of Silicon content of pig iron.	Preparation zone and elaboration zone. The dividing boundary is an isotherm at 1000oC.	Blast moisture, blast temp., and fuel oil injection rate	Inst $P_i =$ Soln. loss $W_u =$ Thes sint comp for m
)	Stabilization of silicon content of pig iron.	Preparation zone and elaboration zone.	Blast humidity	An i mate the fun
	Longitudinal distribution of temp. of gas and solid particles and those of molar fractions of CO and CO_2.	Five: Hearth, Tuyere zone, liquid zone, fusion zone, stack.	-	The of los sto

5	6
$75CO_2 + 1254CO + 2580H_2O_{reqd.} + W_{bd}N_2$ $2 + 0.5(CO_2 + H_2O_{reqd.}) - \beta N_2$ $5042(O_{2ch} + \bar{O}_{2b}) - 1767(C_{cu} + \bar{C}_{add})$ $-441\bar{H}_2O_{reqd} + \bar{W}_b - Q_{add}] / O_{2ch}$	Indices M^g and M^{cu} representing amou of heat input in blast furnace per unit of charge oxygen gasified calcu- lated from top-gas composition and from the composition of charge, blas respectively are closely linked with silicon output of pig iron.
alance $4x_2 + 4.85Z + 3.85W = 67$ l Balance $+1.95\Delta\theta v) + x_2(7.3 + 1.04\Delta\theta v)$ $= 70W + 1610$ $10.8x_2 + 3.1Z = 560$	The blances are made over elaboratic zone. The third eqn. is derived to account for limitation arising from heat requirement of the preparation zone. The model cannot be used for process control.
ntaneous production: $790(A + 640B) - 420$ Carbon $\frac{\Omega}{Z} = \frac{423(B-K)}{P_C} - y$ $\bar{P}_i[533 - 1.72h - (16 - \frac{\Omega}{100})(36.2 + 0.06h) - 324Z$ e equations are for operation with er burden. For the operation with lex material similar eqns can be ulated.	The index Ω represents the differe between heat input and the heat rec uirements of solution loss. It is strongly correlated with the silico content of pig iron. The objective to keep We as close as possible to a set value corresponding to the desired quality of pig iron. This is most widely us ed automatic con scheme for blast furnaces.
ndex Ec is calculated by means of erial and heat balances worked out in elaboration zone. The transfer tion between Ec & Si is $K/(1+TS_1)(1+TS_2)$	The term Ec represents the excess heat available in elaboration zone to reduce silica and to superheat hot metal and slag to the temp. corresponding to the silicon cont of hot metal. It is also correlat with silicon content of pig iron.
reacaion rates of indirect reduction iron ore by CO and one by H_2, s reaction and decomposition of lime- one were taken into account.	In this model the temp. in the individual zones is assumed to b invariant.

furnace is higher than that of the corresponding solids at that level. The furnace was divided into several temperature levels differing by 50°C, and the reactions taking place in these zones were taken into account. The heat required by the burden passing down the furnace was calculated as a function of temperature. The amount of heat available from the gaseous phase in cooling from flame temperature to a given temperature was taken as proportional to drop in the temperature. The resulting curve of heat available versus temperature was nearly a straight line. Hodge et al [16-18] presented a scheme to predict the changes in the furnace performance provided the operating data for the 'standard case' were available. The furnace was divided into three regions, the boundaries of which were defined by fixing the temperatures of solid phase at the boundaries. Three equations were developed, viz., one for heat transfer in the shaft, the second for the amount of carbon consumed in direct reduction and the third for the amount of hydrogen in bosh gas taking part in reduction process. However, if the furnace operating conditions were different from those of the reference case then the model does not hold good. Pokhvisnev et al. [19] divided the furnace into two zones: direct reduction and indirect reduction zones. The heat input into each zone based on one m^3 at NTP of oxygen removed from charge was calculated from top gas composition. The thermal state of the furnace was characterized by the difference between

the current value of heat input and its reference value. Dovgalyuk et al. [20] defined an index ΔM_o representing the change in the heat input to indirect reduction zone in the time of descent of charge from indirect reduction zone into the hearth and was calculated as the difference between the heat input into the furnace as a whole and the heat input into the lower part of the furnace. Wartmann [21] divided the furnace into four zones such as preheating, indirect reduction, direct reduction and hearth. The heat and material balances in each zone were combined with the overall material balance and the quantity of heat transferred from one zone to another, the discharge rate of slag and the rate of consumption of coke were determined. Staib et al. [22-24] pointed out the existence of chemical reserve zone and thermal reserve zone. The chemical reserve zone is the chemically inert zone in which iron exists entirely in the form of wustite. The gaseous phase is such that the ratio per cent CO_2/(per cent CO + per cent CO_2) is 0.238, a value set by the equilibrium: iron-wustite-gas. The thermal reserve zone is one which extends more or less in the shaft where the temperature is periodically constant. The chemical reserve zone lies in the lower part of the thermal reserve zone. Staib divided the furnace into two parts, viz., preparation zone and elaboration zone. The dividing boundary was taken through the chemical reserve zone. The composition and temperature of the gas and the solid phases at the dividing

boundary were fixed by assuming the temperature level of the thermal reserve zone. The thermal state of the blast furnace was characterized by a parameter W_u representing the difference between the heat transferred to solids above 1000°C and heat required for direct reduction per ton of pig iron produced and it was strongly correlated with the silicon content of pig iron. The objective was to keep $W_{\ddot{u}}$ close to a set value corresponding to the desired quality of pig iron. The model is not applicable in cases where the concepts of 'chemical reserve zone' and 'thermal reserve zone' are no longer valid. The model developed by Daconsine [25] consisted of oxygen balance and thermal balance over the elaboration zone. To account for the limitations arising from the heat requirements of the preparation zone, another equation was also included in the model. The model proposed by VanLangen et al. [26-30] was also based on Staib's hypothesis of existence of chemical and thermal equilibrium. The thermal state of the blast furnace was characterized by a factor E_c, representing the difference between the actual heat supplied and the heat required under standard conditions. It was strongly correlated with the silicon content of pig iron. It was calculated by taking heat and material balances over the elaboration zone and was expressed as the excess of heat available in this region to reduce silica and to superheat hot metal and slag above the standard conditions. A control scheme was developed using E_c

as a set point and the blast humidity and blasttemperature as manipulating variables. Flierman et al. [31] divided the furnace into five regions such as hearth, tuyere zone, liquid zone, fusion zone and the stack. Each zone was treated as a stirred tank reactor and the reaction rates of indirect reduction of iron ore by CO and H_2, solution loss reaction and decomposition of lime stone were taken into account. Using this model, the longitudinal distributions of temperatures of solids and gases, and mole fractions of CO and CO_2 were established. The residence time of burden, coke consumption, top gas temperature and its composition were also calculated using this model. Just as Reichardt [14] proposed a graphical model for representing heat balances, Rist et al. [32] presented a method for calculating both mass and heat balances, by means of a diagram known after his name as Rist operating diagram. The diagram illustrates most of the chemical characteristics of the operation: coke rate, reducing gas composition, gas and charge composition at various stages and approach to chemical equilibrium. It is also capable of incorporating heat balances as constraints on the operating line. The effect of variations in various operating parameters like hot blast temperature, oxygen enrichment, natural gas injection, prereduction of the burden were also studied with the help of Rist diagram.

Models Based on Differential Heat and Material Balances: The mathematical models belonging to this group were presented by Koump et al. [33], Lahiri et al. [34] and Muchi et al. [35-36] and have been summarized in Table 2.4.

The model developed by Koump et al [33] divides the blast furnace into three steady-state chemical reactors and one accumulator. The model was developed for Reactor 1 which comprised the region of blast furnace from stack line to an isothermal surface within the stack where liquid phase begins to appear in appreciable quantity. The reactor 1 was treated as a steady-state, one dimensional, countercurrent, heterogeneous, adiabatic reactor in which components in solid phase are reacting with the gaseous phase. The reaction rates of indirect reduction of iron ore by CO and H_2, the solution loss reaction, the rate of heat and mass transfer between the fluid and solid particles, within the solid particles and by bulk flow of gas and solid were taken into account. The model consisted of six ordinary differential equations involving partial pressure of CO and CO_2 in the gaseous phase, mass flow rates of ore and carbon and the temperature of gaseous and solid phases. On the basis of the model proposed by Koump et al., Lahiri et al. [34] presented a model for representing both axial and radial distribution of temperatures of gas and solid particles, mole fractions of CO and CO_2 and the fractional reduction of iron ore in the stack region of the blast furnace, and considered two reactions -

NO. OF REGIONS	REACTIONS CONSIDERED	EQUATI
Four: From stock line to the plane where liquid phase begins to appear; Race ways; and reactor between I and raceways; and on accumulator.	Indirect reduction of Fe_2O_3 to Fe; and gasification of coke. The interchanges of mass and energy were also considered.	
Same as above	Indirect reduction of iron oxide ((Fe_2O_3 - Fe) - Fe) and gasification of coke.	
Same as above	Indirect reduction of iron ore by CO, soluⁿ loss reaction, direct reduction of coustite, decomposition of lime stone, indirect reduction of iron ore by H_2, reaction between coke and steam, water gas shift reaction, reduction of silica by coke.	The model for the zo tuyere level consist differential equatio 3 algebraic equ tion

ASSUMPTIONS	APPLICABILITY
tes of transport of and energy by axial sion are negligible. emp. of solid particle iform throughout and ure is constant throug- eactor. of solid particle is by a single temp. of solid phase.	The longitudinal distribution of partial pressure of reduction of ore and the extent of gasification of carbon in reactor I.
i's & Koump's model I is treated as tate, countercurrent, c, heterogeneous umptions are same as above model.	The longitudinal and radial distribution of temps. of gas and solid particles, molar fractions of CO and CO_2 and fractional reduction of iron ore, in reactor I.
-	The longitudinal distributions of temperatures of gases and solids, fractional reduction of ores, volume rates of flow, composition and densities of gases. The effect of operating variables on coke rate and productivity.

gaseous reduction of iron oxide and gasification of coke. Muchi et al. [35-36] developed a model for blast furnace to calculate production rate, carbon ratio and the longitudinal distribution of process variables in the reactor 1 of Koump. Overall reaction rates of indirect reduction of iron ore by CO and H_2, solution loss reaction, direct reduction of molten wustite by solid coke, decomposition of limestone, reaction between coke and steam, water gas shift reaction and reduction of silica by solid coke were taken into account. Effects of top pressure, diameter of iron ore, volumetric flow rate of blast, ratio of steam injection, blast temperature, ratio of oxygen enrichment and prereduction of iron ore on the productivity and distribution of process variables in the blast furnace under various operating conditions were studied on the basis of this model.

2.1.2 Dynamic Models:

A dynamic model gives the time dependence relationship, that is, the transient behaviour of the process. The development of a control scheme for blast furnace requires a thorough study of the dynamic properties of the furnace. The dynamic characteristics of a physical system introduces one order of complication in a mathematical sense, viz., if a steady state model is described by a set of algebraic equations, the dynamic model will consist of a set of ordinary differential equations

and if steady state model is represented by a set of ordinary differential equations then the dynamic model will consist of a set of partial differential equations. For investigating the dynamic characteristics of a blast furnace the method of approximation to 'lumped parameter' system is often adopted. A large volume element which can be assumed to be completely mixed is selected as a cell instead of infinitesimal element to decrease the mathematical complexity. Based on this approach, dynamic models were proposed by Fielden et al. [37-40], Beer et al.[41], Horio et al.[42] and Tsuchiya et al.[42].

Fielden et al. [37-40] divided the blast furnace into five regions, namely, stack, upper bosh, lower bosh, tuyere and hearth . The furnace was further subdivided into zones of one meter height, each acting as a batch reactor. The state of each zone was specified in terms of composition and temperature of the burden material and gas. A state matrix was defined, each row of which described the contents of each zone. It was assumed that the gaseous phase and burden material stay in each zone for a definite time interval, during which they react and move instantaneously to the next zone. An estimate of the top gas analysis and temperature was used by the authors to update the coefficients of the model. The model was used for automatic regulation of blast conditions and charge in order to eliminate cast-to-cast fluctuations in the silicon content of pig iron. Beer et al. [41] followed a procedure similar to that of

Fielden et al., by dividing the furnace into n volumes of the same size in which physical and chemical changes were determined by a system of differential equations obtained from heat and mass balances. Horio et al. (see [42]) represented the furnace region below melting zone by 'tank-in-series' model and investigated response of lower region of blast furnace to a step change of blast conditions. The fluctuations of 'heat level' were connected with the magnitude of the coke reserve. In the model developed by Tsuchiya et al. (see [42]), the furnace was divided into five regions, such as, region 1 in which only heat exchange between ascending gas and descending burden material takes place, region 2 in which reduction of Fe_2O_3 to Fe_3O_4 takes place, region 3 in which reduction of Fe_3O_4 to FeO takes place, region 4 in which gaseous reduction of FeO as well as solution loss reaction take place and region 5 in which combustion of coke takes place. All these regions were assumed to act as stirred tank reactors. A steady state was assumed with respect to mass balance and with respect to heat balance in the gaseous phase. Considering the temperature to depend only on the solution loss reaction, the fluctuations of solid temperature in region 5 was regarded as a criterion of 'heat level' and was calculated from observed top conditions. This was compared with the fluctuations of silicon content of tapped metal. Based on a dynamic heat balance, Hodge et al.[43] defined three additional variables to supplement the theoretical

flame temperature for controlling hearth heat, namely, total heat generated, heat available above 2800^{o}F and the ratio of heat available above 2800^{o}F to the total heat input. The effect of variables like blast temperature, blast humidity, oxygen enrichment, fuel oil injection and natural gas injection on these three quantities were determined. The authors have suggested two more thermal ratios as possible criteria, namely, the total heat input divided by the burden charged and the heat available above 2800^{o}F divided by the burden charged. It was pointed out by Bouman [44] that these thermal ratios do not deal with changing reducing requirements in the hearth and stack. It is well known that even with constant blast conditions and without apparent changes in burdening, thermal conditions are subject to variation, which is indicated by iron analysis and its temperature. The change in burdening cannot be detected prior to its causing a thermal upset in the hearth.

Aerodynamic Models: Bates [45] developed an aerodynamic model of the blast furnace relating raw material properties to furnace productivity. First, by utilizing classical packed bed theory certain relationships like pressure drop, surface area etc. were developed. A set of equations relating material geometry of the burden materials to the permeability were then developed. The material geometry changes inside the furnace were related to position inside the furnace, operating conditions

and to the different types of material. The furnace was divided into five zones, such as, charging zone, indirect reduction zone, melting zone, molten zone and turbulent zone and in each zone the average values for material parameters were used to describe that zone. Klempert et al. [46] developed an aerodynamic model for the blast furnace which can calculate following process indices: coke retention time in the furnace, height of zones of complete and partial melting of materials, average lump size for each zone, average size of iron ore burden and fine formation in the charge as it descends in the region of solid materials and the gas stream temperature up the furnace. The furnace was divided into four zones. The movable boundaries between zones were calculated by successive approximation on the basis of material and heat balances for one hour of furnace operation. The stack column voidage for different zones was calculated with allowance for size degradation of sinter lumps and their gradual melting as they descend.

2.2 PROBABILISTIC MODELS:

In probabilistic models, the technique of statistical analysis is used for describing a physical system. The probabilistic models are broadly classified as regression models, stochastic linear process models and time series models. A summary of probabilistic models has been given in Table 2.5.

WORKER	TYPE OF MODLE	AIM	PRINCIPLE	
Flint (1952)	Stationary	Coke rate	Multiple regression analysis	Coke r.
Kudoyarov et al. (1967)	Stationary	Productivity & coke rate	Multiple regression analysis	K = 0. + 6 - 0 P = 0. - 8 - 0
Starshinov (1968)	Stationary	Productivity & coke rate	Multiple factor correlation analysis	C = 71 P = - +
Kobylakov et al. (1971)	Stationary	Productivity & coke rate	Multiple factor correlation analysis	K = - + P = - +
Robbins (1969)	Stationary	Blast Temperature	Stepwise regression analysis	T_b =
Katsura (et al) 1965)	Dynamic	Silicon content	Time series analysis & regression analysis	Z $\triangle$ S ΔS_{n}
Spallanzani (1965)	Dynamic	Silicon content & pig iron temp.	Regression analysis	Si_n = T_{Gi} =
Norton (1973)				

EQUATIONS	REMARKS
as a linear function efficients	It is applicable by management for long-term planning and by plant operators for short-term control. It cannot be extrapolated or generalized to new operating conditions.
23Si − 0.253T − 8Pt + 0.225W .327S − 12.5Fe + 2.18Ac − 0.172F .25L + 1170 67Si + 0.0713T + 9Pt − 0.647W .36S + 24Fe − 4.6Ac − 0.245L 98	The method can be used to evaluate effect of various parameters on the blast furnace performance. Discrepanly : ± 1%
4[Si] − 0.188tb + 0.145L + 902 − 0.0004348tb − 0.0001715L	The method enables the dependence of blast furnace operating indices on parameters to be found & the significance of their influence as a whole and individually to be established.
$36x10^{-4} G^2 + 1.638G - 0.214tb + 4.236V_b + 7182$ $58G^2 + 38.5G - 113x10^4/tb$ 7283	The discrepancy : ± 1.0 − ± 1.5, The parameter values at which best blast furnace operating indices are obtained can be determined.
$.0V_b - 10.7\frac{C}{O} + 10.5\frac{Si}{O} + 49.3CO + 38.1H_{loss}$	The method is used to control heat balance by controlling blast temperature
$+ b_2F_bT_b + b_3F_h + b_4F_bM + b_6Si_{n-1}Z + b_7Z + b_8$	The difference between heat input & output was assumed to be proportional to the difference between Si content in pig iron & that of preceding tapping.
$Si_{n-1} + C_2\lambda_n + C_3(\alpha + \beta_{n-1} + V_{n-1}) + M \frac{F_b}{P_b - P_t}) + C_6$	Controllable variable : Blast humidity.
$+ C_2O_{RI} + C_3O_{RD_1} + C_4O_{RD_2} + C_5T_{V_1}$ $_7Si_{n-2} + C_8Si_{n-1}$ $Si_{i-1}, Mn_{i-1}, \Delta Si$)	The selection of factors was strictly related to operating conditions. This represents first limitation to the possibility of generalizing the formulas.

2	3	4	5	
a et.al. 1970)	Stationary	Controlling heat level of the lower zone.	Automatic correlation method.	Blast humidity was $M = \bar{M}_n + (q/_o - \bar{q}/_o$
s 52)	Stationary	Production rate.	Regression analysis	MTR $= C_5 + C_1 T_B + C_2$ $+ C_6 (st/0) -$ $= f(c_j, x_j)$ P $= \frac{k f_1 (c_j, x_j)}{1 + c/0 + st,}$

I	7
by the equation : $\alpha_1-\bar{\alpha}_n)+q/_2([S_i]-[\bar{S}_{i_n}])$	All burden and blast conditions except blast humidity were kept constant. Humidity was used as maltipulating variable.
$+C_4 P_J + C_5(C/o)$ ore. L	The constraints on sulphur and silicon content of pig rion, slag volume, slag basicity , hearth temperature, top temperature and temperature probe were made. And optimum operating conditions at steady - state were found by masimizing the objective function.

2.2.1 Regression Models:

Two approaches have been used in the past to develop regression model for the blast furnace. While the first approach gives the production rate and coke rate through regression models on concurrent values of other variables, the second approach gives output variables taking time lag into account.

For constructing a regression model for operating indices (productivity and coke rate), only the characteristics of the materials charged and the quality of end products are to be considered. With this approach the blast furnace represents a 'black box' in which only inputs and outputs are considered without knowing the basic mechanism of the process. Such models are helpful in production planning and control. However, these models do not give insight to the process and are unsuitable for extrapolation and for applying to other similar systems. Therefore, these models are, at best, suitable to a particular set of operating conditions of a given system and outside this range new models may have to be developed. The technological parameters of the process that are usually considered for regression model are - blast volume, blast temperature blast humidity, consumption of natural gas, oxygen enrichment, fuel oil injection, operating intensity, composition of pig iron, slag volume, iron content of burden, basicity ratio, proportion of sinter in the burden, composition of lime stone,

coke rate, sinter-to-coke ratio and top pressure etc. Models based on multiple regression analysis were developed by Flint [47-48], Kudoyarov [49], Starshinov [50] and Kobylyakov etal. [51] The models developed by Flint, Kudoyarov and Starshinov were linear in parameters whereas Kobylyakov et al. suggested a non-linear model. Flint [47-48] expressed coke rate as a linear function of 21 variables. Kudoyarov [49] suggested that the coke rate and productivity were most affected by the composition of burden, and silicon content of pig iron. He found that an increase in the blast humidity lowered the productivity and increased the coke rate. Starshinov [50] found that for an increase in the silicon content coke rate increases and productivity decreases. Increase in the operating intensity increases both coke rate as well as productivity. Kobylyakov et al. [51] developed non-linear relationships between operating indices and technological parameters through dispersion and correlation analysis and obtained optimum operating parameters.

The models based on regression analysis with lagged variables were developed by Maeda et al. [52], Robbins [53], Katsura et al. [54], Spallanzani et al. [55], and Norton [56].

Maeda et al. [52] developed an equation relating blast humidity with silicon content of pig iron, which was used for controlling thermal state of the blast furnace. The blast humidity was expressed as a function of amount of direct reduction which was calculated from top gas analysis. Robbins[5

expressed set point for blast temperature as a function of blast moisture, wind rate, top gas temperature, carbon monoxide content of top gas and heat losses to cooling water. The model developed by Katsura et al. [54] was based on the fact that the difference between the silicon content of pig iron and that of preceding tapping is directly proportional to the heat storage in the lower part of the furnace. This difference of the silicon content was related to the variables like blast temperature, humidity, flow rate; amount of direct and indirect reduction; amount of heterogeneous reaction and permeability in the furnace. Spallanzani et al. [55] assumed a hypothesis similar to that of Katsura et al. and predicted the silicon content and hot metal temperature of next cast based on variables measured at 9 and 5 hours before the expected casting time. While Robbins had completely ignored the time lags (between the charge and the cast and between the blast and the cast), Katsura et al. have considered the time lag between charge and the cast and Spallanzani et al. have considered both the time lags and constructed the model by delaying certain measurements. Norton [56] has developed a model in which an output variable at any time is expressed as a linear function of the input and output variables at previous sampling times and a noise. The equation was linear but the variables which had non-linear relationship with the measured variables were separately estimated and used in the linear regression. The linear models

were developed for silica reduction rate and production rate by linearization about the normal operating conditions. The production rate model was based on material balance and the silicon reduction rate model was based on the heat balance above 1000°C.

2.2.2 Linear Stochastic Process Models:

Another group of the probabilistic models is the linear stochastic process model. The input and output variables of the blast furnace are subject to random perturbations that can be modelled by probabilistic laws and hence a linear stochastic process model can be used to relate input and output variables.

A steady-state is first carefully established. A specified perturbation (impulse, step or sinusoid) can be given to an input variable and the response of the system on one or more output variables may be measured. They are used in estimating the system parameters. This is an 'instrumentation problem'. Luckers et al. [57-58] used binary perturbations, mainly square waves to estimate coefficients of various transfer functions, whose forms were chosen from previous experience. Models relating blast temperature, blast humidity, oil injection rate and coke-to-sinter ratio to the silicon content of the hot metal and the parameter E_c were fitted. After logging the furnace response at a sampling interval of 20 minutes the cross-correlation function between input and

output was computed. The plant model was then fed with a waveform representing the autocorrelation function of the perturbation input and the coefficients of the transfer function were adjusted until a satisfactory fit was obtained between the model output and the recorded cross-correlation function. The Wiener-Hopf equation was thereby used in which knowing the autocorrelation function of the input,the impulse response was adjusted until best input-output cross-correlation function was obtained. It was found that silicon response to blast temperature variations and changes in the flow of fuel oil is very slow. The transfer function between blast humidity and silicon content had shorter time constant but its form was'inverse response type'[58]. The silicon response to the coke-to-sinter ratio included a dead time in addition to the time constants. Staib et al. [59] have also studied the effect of step response of fuel oil rate on hot metal silicon content. Rebeko et al.[60] also used Wiener-Hopf equation to determine the impulse response function knowing the input-output cross-correlation function and the input autocorrelation function. The mathematical model was based on calculation of available heat in the direct reduction zone at a series of successive moments of time after a step wise change in ore-to-coke ratio. The system transfer function was obtained from the impulse response function. The transfer functions between the variables like ore-to-coke ratio, iron content in sinter, oxygen content of the blast, proportion

of natural gas in the blast and silicon content of pig iron were determined.

The classical methods of estimating transfer functions based on deterministic perturbations, however, have not always been successful because of the following reasons: (i) These tests require carefully controlled furnace conditions for several days per test and create considerable operating inconveniences. Similarly the sequence of pseudo-random binary signals would each require several days of data logging, during which a few missing or unreliable points or necessary control action would spoil the experiment. (ii) The blast furnace is subjected to a large number of disturbances which cannot be taken into account in such a model. (iii) It is difficult to decide on the amplitude of variations to be imposed so that it should take into account, on one hand, the size of the disturbance and on the other hand the upper limit imposed by the need to remain close to the normal operating conditions. Some of these limitations are overcome in time series models.

2.2.3 Time Series Models:

Time series models were developed by Rylov [61], Rake [56], Rebeko et al. [60], Castore et al. [62] and Phadke et al. [2].

Rylov [61] applied the methods of statistical dynamics to study the relationships between rate of descent of charge

material with the input quantities - blast volume, temperature, humidity, and natural gas consumption. On the basis of cross-correlation coefficient it was shown that the relationship between the rate of descent of charge and blast volume had an extremum. A system of extremal regulation for the rate of descent of charge by changing the blast volume was proposed. Rake (see [56]) determined the transfer function at a given frequency between the heat index analogous to the parameter $W_{ü}$ and the silicon content of pig iron from the power spectrum of the input and the cross-spectrum between the input and output. These spectra were calculated by taking Fourier transformation of autocorrelation function and input-output cross-correlation function. Rebeko et al.[60] used the methods of statistical dynamics and the theory of random functions for predicting thermal state of the blast furnace, particularly silicon content in the next cast, fromthe information on the current operation of the furnace. On the basis of primary information for a definite time interval, the autocorrelation and cross-correlation functions between input and output variables were determined. The position of cross-correlation maxima was determined and the coefficients of multiple regression equation, between the parameters taken with appropriate time displacements were determined. Castore et al. [62] have developed a feed-back control scheme for hot metal silicon content using blast humidity, blast temperature and ore-to-coke ratio as

manipulating variables. The input variables were subjected to programmed stepwise variations and transfer functions between these independent variables and silicon content were developed. The disturbance at the casting frequency was identified by means of statistical analysis of time series of finite length collected during the periods of normal conditions. In the feed back loop, the output of the 'plant model' after sampling and delaying, and the disturbance which represented the variable component of silicon content in open loop, were fed to the controller which determined the correction on the control variable. Phadke et al. [2] have developed a multivariate time series model for the blast furnace, taking into account two input variables (blast flow rate and ore-to-coke ratio) and three output variables (sulphur content, silicon content and temperature of molten metal). Using this multivariate model (described in detail in Chapter 4), they have also derived various transfer functions, viz., for the plant, for the feedback system, for the blast furnace noise and for the feed back noise.

Because of the advantages of time series modelling over other types of models for the blast furnace process, it is proposed to adopt time series modelling in this study. In particular, Phadke's model is adapted and used.

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CHAPTER 3

UNIVARIATE TIME SERIES ANALYSIS

3.1 INTRODUCTION:

A time series is defined as a set of observations made sequentially in time. If the set is continuous, the time series is said to be continuous. If the data are given for discrete values of time as sampled or quantized data, the time series is referred to as a discrete time series. If the future values of a time series are exactly determined by a mathematical function using the present and the past values, the time series is said to be deterministic. If the future values can be described only in terms of a probability distribution, the time series issaid to be a non-deterministic or a stochastic time series.

A time series is said to be strictly stationary if the joint probability density function of the families of the variable $[z(t_1), \ z(t_2), \ldots, z(t_n)]$ and $[z(t_{1+i}), \ z(t_{2+i}), \ldots, z(t_{N+i}]$ is the same for all i for any set of $t_1, \ t_2, \ldots, t_N$; **ie., stationarity** is achieved if the distribution is invariant with respect to translation in time. This implies stationarity of moments of all orders of the time series. First order stationarity implies that mean is constant and second order stationarity implies that the autocovariance at lag k, viz.,

$E[z(t) \cdot z(t+k)]$, where E stands for the expectation, is a function of k only. This also implies that the variance of z(t) is constant. Generally a second order stationarity is assumed.

A time series can generally be considered to consist of deterministic components that include trend and cyclicity; a non-random component which includes peristence, and a pure random component. Trend refers to long-term behaviour of the time series and it may indicate a monotonous change in the main value - which can be expressed as a linear, polynomial or exponential function of time, and/or jumps in the mean value. A plot of data may indicate whether a trend is present. Cyclicity refers to the periodic components that repeat themselves at definite intervals. Persistence refers to the linkage between the value at a given time with earlier values and may be due to internal or external dependence. A time series may be represented in terms of its several components as follows:

$$z(t) = f_{trend}(t) + f_{cycle}(t) + f_{AR}(z(t-1), z(t-2), \ldots, z(t-p)) + f_{MA}(\varepsilon(t), \varepsilon(t-1), \ldots, \varepsilon(t-q)) + \varepsilon(t) \tag{3.1}$$

in which $\varepsilon(t)$ represents the pure random component of the time series and f_{AR} and f_{MA} represent autoregressive and moving average components, representing respectively the internal and external dependence in the time series. The random component

is the residual series obtained after subtracting trend, cycle, and autoregressive and moving average components from the original time series. Very often some components may not be present and the resulting time series is simpler.

3.2 TIME SERIES MODELS:

An important class of stochastic models for describing a time series is the so called stationary models. A time series is said to be stationary if the generating mechanism of the process is invariant with time. This implies that the deterministic component of the series is independent of time and the parameters of the model are also independent of time.

3.2.1 Sum of the Harmonic Process Model:

In order to identify deterministic components of a time series, it can be decomposed into a number of sinusoids of varying frequencies and amplitudes. When the data are finite and discrete with $N=T/\Delta t$ samples, where T is the time horizon and Δt is the time interval, z(t) can be approximated by a finite Fourier series that passes through all sample points, viz.,

$$z(t) = \sum_{i=1}^{M} c_i' \sin(w_i t + \theta_i') + x(t) \qquad (3.2)$$

where c_i' represents amplitude, θ_i' is the phase angle and w_i is the angular frequency given by

$$w_i = 2\pi f_i = 2\pi i/T \qquad (3.3)$$

in which $f=1/T$ is the fundamental frequency and i represents the ith harmonic. If the shape of the time series is that of a sine curve then the entire fluctuations of the series would be contained in one harmonic. When the shape of function is not sinusoidal, then atleast two and atmost $N/2$ harmonics are required to approximate a function through all sample points.

3.2.2 Autoregressive Model:

This is a linear model for persistence within the time series, in which the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a random component. It is given by

$$\bar{x}(t) = \varphi_1\bar{x}(t-1) + \varphi_2\bar{x}(t-2)+\dots+\varphi_p\bar{x}(t-p)+ \varepsilon(t) \tag{3.4}$$

where $\bar{x}(t)$ represents the deviation from the mean μ of the original time series $x(t)$ and $\varepsilon(t)$ is a random component of mean zero. This is an autoregressive process of order p or AR(p) process. The backward shift operator B is defined by

$$B\bar{x}(t) = \bar{x}(t-1) \quad \text{and} \quad B^k\bar{x}(t) = \bar{x}(t-k) \tag{3.5}$$

Then Eqn. 3.4 can be written as

$$\varphi(B)\ \bar{x}(t) = \varepsilon(t) \tag{3.6}$$

where $\varphi(B)$ is the autoregressive operator of order p defined by

$$\varphi(B) = 1-\varphi_1 B - \varphi_2 B^2-\dots-\varphi_p B^p \tag{3.7}$$

Let σ_ε^2 represents the variance of the white noise process $\varepsilon(t)$, then the model contains p+2 unknown parameters, μ; $\varphi_1,\varphi_2,\dots\varphi_p$;

and σ_ε^2 which have to be estimated from the data.

3.2.3 Moving Average Model:

This is a model indicating external correlation between the time series x(t) and a pure random time series ε(t). In this model, the current value of the process is expressed as a linear function of the present and the previous values of the random component ε(t), and is given by

$$\bar{x}(t) = \varepsilon(t) - \theta_1\varepsilon(t-1) - \theta_2\varepsilon(t-2) - \ldots - \theta_q\varepsilon(t-q) \quad (3.8)$$

This is a moving average process of order q or MA(q) process. Equation 3.8 can also be written in terms of backward shift operator, B, as

$$\bar{x}(t) = \theta(B)\ \varepsilon(t) \quad (3.9)$$

where $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots\ldots - \theta_q B^q$ (3.10)

is the moving average operator of order q. The model contains q+2 unknown parameters μ; $\theta_1, \theta_2, \ldots, \theta_q$ and σ_ε^2 which have to be estimated from data.

3.2.4 Autoregressive - Moving Average Model:

To achieve greater flexibility in fitting a time series model, it is sometimes advantageous to include both the autoregressive and moving average components in the model. This leads to a mixed or autoregressive-moving average(ARMA) model of order (p,q):

$$\bar{x}(t) = \varphi_1\ \bar{x}(t-1) + \varphi_2\bar{x}(t-2) + \ldots + \varphi_p\bar{x}(t-p) + \varepsilon(t) - \theta_1\varepsilon(t-1) \ldots\ldots - \theta_q\varepsilon(t-q) \quad (3.11)$$

which can also be written as

$$\varphi(B)\ \bar{x}(t) = \theta(B)\ \varepsilon(t) \tag{3.12}$$

It contains p+q+2 unknown parameters that have to be estimated from the data.

3.2.5 <u>Autoregressive-Integrated-Moving Average Model</u>:

The above models are stationary with respect to mean and standard deviation and they also assume covariance stationarity. A more general class of models is one which is non-stationary and whose stationarity can be achieved by repeated differencing of the original series. Let

$$w(t) = \nabla^{d}\ x(t) \tag{3.13}$$

where ∇ is the backward shift operator defined as

$$\nabla x(t) = x(t) - x(t-1) \tag{3.14}$$

and d is the order of differencing. A stationary ARMA(p,q) model is then fitted to the differenced series w(t). The original series x(t) is said to be autoregressive-integrated-moving average(ARIMA) model of order (p,d,q) where d indicates the order of differencing required to render the original process stationary.

In general, the time series can be expressed by the general ARIMA model in which p and q are not greater than 3 and d is not greater than 2.

3.3 LINEAR SYSTEMS:

A system, whose behaviour can be described by a linear differential or difference equation is said to be linear.

3.3.1 Linear Differential Equation:

A continuous linear process with input $\varepsilon(t)$ and output $\bar{x}(t)$ can be represented by a linear differential equation with constant coefficients, as follows:

$$[a_o+a_1D + a_2D^2+\ldots+a_pD^p]\ \bar{x}(t) = [b_o+b_1D+b_2D^2+\ldots+b_qD^q]\varepsilon(t) \tag{3.15}$$

where a's and b's are constants and D is the differential operator. A system characterized by a differential equation is dynamical, because the value of the output $\bar{x}(t)$ at any instant t depends not only on the value of $\varepsilon(t)$ at that instant but also on the values of the derivatives of $\varepsilon(t)$ and $\bar{x}(t)$ at that instant which, in turn, are dependent on the values of $\bar{x}(t)$ and $\varepsilon(t)$ at other instants. A system described by Eqn.3.15 is time invariant if the differential equation relating the input and output signals has constant coefficients.

3.3.2 Linear Difference Equation:

Just as in the case of a continuous system, many properties of a discrete system are exhibited in the difference equation relating the input and output signals of the system. A discrete linear system with input $\varepsilon(t)$ and output $x(t)$ may be represented by a linear difference equation with constant

coefficients, as follows:

$$c_0\bar{x}(t) - c_1\bar{x}(t-1) - c_2\bar{x}(t-2).....-c_p\bar{x}(t-p) =$$

$$v_0\varepsilon(t) - v_1\varepsilon(t-1) - v_2\varepsilon(t-2)....v_q\varepsilon(t-q) \qquad (3.16)$$

where c's and v's are constants.

A discrete system described by Eqn. 3.16 is dynamical because the value of $\bar{x}(t)$ depends not only on the values of $\varepsilon(t)$ but also on the values of $\bar{x}(t-1)$, $\bar{x}(t-2)$,....,$\varepsilon(t-1)$, $\varepsilon(t-2)$,..... The system described by Eqn. 3.16 is also time invariant if the coefficients of the difference equation are all constants. The general ARMA model given by Eqn. 3.11 is exactly similar to the linear difference equation given by Eqn. 3.16. It can be seen that there is close resemblance between the linear differential equation with constant coefficients and the ARMA model.

3.3.3 Stability:

A system is said to be stable [63], if the magnitude of the output is bounded at all times when the magnitude of the input is bounded at all times. A system which generates the time integral of its input need not be stable since the magnitude of the output can be unbounded even when the magnitude of the input is bounded. A discrete system whose output is the forward or backward difference of its input is always stable. Since a pure moving average process, described by Eqn. 3.8 can be considered as an output from a linear system

whose input is a white noise, it is always stable.

Stability of Continuous System: The characteristic equation of the differential equation 3.15 is as follows:

$$a_p s^p + \dots + a_1 s + a_o = 0 \qquad (3.17)$$

The system is said to be stable if the roots of the characteristic equation have negative real parts.

Stability of Discrete System: Let z be a shift operator with the property

$$z^{-k}x(t) = x(t-k) \qquad (3.18)$$

Then the difference equation 3.16 may be written as

$$(c_o - c_1 z^{-1} - c_2 z^{-2} - \dots - c_p z^{-p})\, \bar{x}(t) =$$
$$(v_o + v_1 z^{-1} + v_2 z^{-2} + \dots + v_q z^{-q})\, \varepsilon(t) \qquad (3.19)$$

thus

$$\bar{x}(t) = \frac{v_o + v_1 z^{-1} + v_2 z^{-2} + \dots + v_q z^{-q}}{c_o - c_1 z^{-1} - c_2 z^{-2} - \dots - c_p z^{-p}}\, \varepsilon(t) \qquad (3.20)$$

The characteristic equation of a discrete system is obtained by equating the denominator of Eqn. 3.20 to zero. Thus

$$c_o z^p - c_1 z^{p-1} - \dots - c_p = 0 \qquad (3.21)$$

The system is stable if the roots of the characteristic equation lie inside the unit circle in z plane.

3.4 ANALYTICAL PROCEDURES:

The periodic and persistence components of the time series are identified by correlation analysis and spectral analysis. The autocorrelation function, partial autocorrelation function and spectral density function are important tools in identifying the periodic and persistence components. These are helpful not only to identify the form of the model but also to obtain approximate estimates of the parameters.

3.4.1 Autocorrelation Function:

The autocorrelation function determines the linear dependence among successive values of a series that are a given lag apart. The autocovariance function between two values z(t) and z(t+k) of a time series that are tk lags apart is given by

$$\gamma(k) = \mathrm{cov}[z(t)\cdot z(t+k)] = E[(z(t)-\mu(t))(z(t+k)-\mu(t+k))] \tag{3.22}$$

The autocorrelation function, ρ_k at lag k is given by

$$\rho_k = \frac{\mathrm{cov}[z(t)\cdot z(t+k)]}{[\mathrm{var}z(t)\cdot \mathrm{var}z(t+k)]^{\frac{1}{2}}} = \frac{E[(z(t)-\mu(t))(z(t+k)-\mu(t+k))]}{\sqrt{E[(z(t)-\mu(t))^2]E[(z(t+k)-\mu(t+k))^2]}}$$

$$= \frac{\gamma_k}{\gamma_o} \tag{3.23}$$

The plot of the autocorrelation function with lag is known as a correlogram.

Estimation of Autocorrelation Function: In practice, from a finite sample of the time series one can only obtain the estimate r_k, of the autocorrelation ρ_k, where

$$r_k = \hat{\rho}_k = \frac{R_k}{R_o} \qquad (3.24)$$

in which $R_k = \frac{1}{N-k} \sum_{t=1}^{N-k} (z(t)-\hat{\mu})(z(t+k)-\hat{\mu})$

is the estimate of autocovariance γ_k. For computational purposes, the autocorrelation function may be calculated from

$$r_k = \frac{\frac{1}{N-k} \sum_{i=1}^{N-k} z(i)\, z(i+k) - \frac{1}{(N-k)^2}\left[\left(\sum_{i=1}^{N-k} z(i)\right)\cdot\left(\sum_{i=1}^{N-k} z(i+k)\right)\right]}{(\text{var } z(i)\cdot\text{var } z(i+k))^{\frac{1}{2}}} \qquad (3.25)$$

where $\text{var } z(i) = \frac{1}{N-k} \sum_{i=1}^{N-k} z(i)^2 - \frac{1}{(N-k)^2}\left[\sum_{i=1}^{N-k} z(i)\right]^2$

<u>Standard Error ofEstimated Autocorrelation Function</u>: The estimated value of the autocorrelation function differs from the theoretical value because of sampling errors and finite sample size. Hence, it is important to have some indication of how far an estimated value may differ from the theoretical value. In order to test that the autocorrelations are not significantly different from zero after some log q, Bartlett[64] has suggested the following equation for the standard error of estimated autocorrelations,

$$\hat{\sigma}[r_k] \simeq \frac{1}{N^{\frac{1}{2}}} \left[1+2(r_1^2 + r_2^2+\ldots+r_q^2)\right]^{\frac{1}{2}}, \qquad k > q \quad (3.26)$$

so that all autocorrelations beyond $\pm$ 1.96 times the standard error may be considered significant at 95 per cent confidence level.

3.4.2. Partial Autocorrelation Function:

The partial autocorrelation function φ_{kk} is useful in identifying particularly the order of an autoregressive process. The partial autocorrelation coefficients are related to the autocorrelation coefficients by Yule-Walker equations given by[see 65,66]

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_p \end{bmatrix} = \begin{bmatrix} 1 & \rho_1 \cdots\cdots & \rho_{p-1} \\ \rho_1 & 1 \cdots\cdots & \rho_{p-2} \\ \vdots & \vdots & \vdots \\ \rho_{p-1} & \rho_{p-2} & 1 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_p \end{bmatrix} \qquad (3.27)$$

If the order of the process, p, and the autocorrelation functions, $\rho_1, \rho_2, \ldots \rho_p$ are known, the above system of p equations with p unknowns $\varphi_1, \varphi_2, \ldots, \varphi_p$ can be solved. In practice, the true values of p and ρ_i are unknown. Let p=1, Using Yule-Walker equations and using the estimated value r_1, for ρ_1 one gets $r_1 = \hat{\varphi}_1$, where $\hat{\varphi}_1$ is the estimate of φ_1. If $\hat{\varphi}_1$ is significantly different from zero, it can be concluded that the process is atleast of order one. To see whether the process is of order two or greater, the Yule-Walker equations are solved for p=2, i.e.,

$$\begin{aligned} r_1 &= \hat{\varphi}_1 + \hat{\varphi}_2 r_1 \\ r_2 &= \hat{\varphi}_1 r_1 + \hat{\varphi}_2 \end{aligned} \qquad (3.28)$$

in which r_k stands for the estimate of theoretical autocorrelatio[n] coefficient, ρ_k.

If the resulting estimate of φ_2 differs significantly from zero, it can be concluded that the process is atleast of order 2. This procedure is repeated successively for larger values of p. If the true order of model is p_{true}, then, when the system is solved for $p=p_{true}+1$, the value of $\hat{\varphi}_{p_{true+1}}$ will not be significantly different from zero since it is an estimate of $\varphi_{p_{true+1}}$, which is zero. Denoting by $\hat{\varphi}_{ii}$, the value of $\hat{\varphi}_i$ obtained by the solution for p=i, $\hat{\varphi}_{ii}$ are referred to as the estimated partial autocorrelation coefficients of the process. If the order of autoregression is p_{true}, then

$$\hat{\varphi}_{ii} = 0 \text{ for } \quad i > p_{true} \tag{3.29}$$

The partial autocorrelation coefficients are calculated using Eqn. 3.30

$$\hat{\varphi}_{ii} = \begin{cases} r_1 & i=1 \\ \dfrac{r_i - \sum\limits_{j=1}^{i-1} \hat{\varphi}_{i-1,j}\, r_{i-j}}{1 - \sum\limits_{j=1}^{i-1} \hat{\varphi}_{i-1,j}\, r_j} & i=2,3,\ldots. \end{cases} \tag{3.30}$$

where $\hat{\varphi}_{ij} = \hat{\varphi}_{i-1,j} - \hat{\varphi}_{ii}\,\hat{\varphi}_{i-1,j-1} \qquad j=1,2,\ldots i-1$

Standard Error of Estimated Partial Autocorrelation Coefficients:

Since the partial autocorrelation coefficients are sample statistics and therefore subject to sampling error, a test is needed to decide when $\hat{\varphi}_{ii}$ is indistinguishable from zero in a statistical sense. Quenouille [6 7] has shown that on the

hypothesis that the process is autoregressive of order p, the estimated values of order p and higher are approximately normally distributed with standard deviation

$$SE[\hat{\varphi}_{kk}] \simeq 1/\sqrt{N} \qquad k \geqslant p+1 \tag{3.31}$$

Thus it is inferred that $p = p_k$ at 95 per cent confidence level if $\varphi_{p_k+1,\ p_k+1}$ is small compared to $\pm\ 1.96/\sqrt{N}$.

3.4.3 Spectral Density Function:

Spectral analysis is useful in identifying periodic and persistence components of the time series. For discrete data the frequency domain representation of z(t) is a discrete line spectrum. A complex periodic data consist of a number of sinusoids with amplitudes c_i' , and phases θ_i' [see Eqn. 3.2]. The plot of amplitude versus frequency is called a periodogram. The spectral density function is a normalized periodogram and it shows how variance of z(t) is distributed over the frequencies. It is obtained by taking Fourier transformation of the auto-covarance function. If M represents the maximum lag (usually 10 per cent, of the number of data points) upto which auto-covarance, R_k, is computed, then thesample spectral density function $\tilde{G}_i$ is estimated by

$$\tilde{G}_i = \tilde{G}(f) = \frac{2}{M}\left[R_o + 2\sum_{k=1}^{M-1} R_k \cos\frac{\pi\, k\, f}{f_c} + (-1)^M R_M\right] \tag{3.32}$$

where f is the frequency given by

$$f = \frac{i}{2M\Delta t}, \qquad i = 0, 1, 2, \ldots, M \tag{3.33}$$

The cut-off frequency, $f_c = 1/(2\cdot\Delta t)$ is the highest frequency cycle that can be identified from a given discrete series with time interval Δt. The spectral density function $\tilde{G}(f)$ has a negative exponential distribution and so the sample value $\tilde{G}(f)$ may be widely different from the true population value. This is because, while the sample autocovariance function is a good estimate of the point value, the simultaneous estimates for all points are not good estimates. It is difficult to identify the components of the time series from the raw spectrum. Hence, it is necessary to smoothen the raw spectrum. There are several procedures available for this purpose. One way is to compute the spectrum and then perform the smoothing operation. Another way is to multiply the autocovarance function by a weighting function, and then compute the spectrum. In this study, a method suggested by Blackman and Tukey[see 68] is used, viz., smooth spectra G_i are given by

$$\begin{aligned} G_o &= 0.54\,\tilde{G}_o + 0.46\,\tilde{G}_1 \\ G_M &= 0.54\,\tilde{G}_M + 0.46\,\tilde{G}_{M-1} \\ G_k &= 0.23\,\tilde{G}_{k-1} + 0.54\,\tilde{G}_k + 0.23\,\tilde{G}_{k+1} \qquad 1 < k < M \end{aligned} \tag{3.34}$$

3.5 STEPS IN FITTING A UNIVARIATE TIME SERIES MODEL:

There are three steps in fitting a time series model, viz.,

(i) Identification: Using the data the order of differencing and the form of the model are determined.

(ii) Estimation: Initial (approximate) parameter estimates are obtained from autocorrelation function and are used as starting values. The parameters of the identified model are then estimated using more refined procedures.

(iii) Validation of the Model: The residuals from the fitted model are subjected to diagnostic checking to test the adequacy of the model. If the residuals are purely random, no lack of fit is indicated, and the model is considered to represent the physical system. If any inadequacy is found, the iterative cycle of identification, estimation and diagnostic checking is repeated until a suitable representation is found.

3.5.1 Data Used for Study:

Pig iron is produced in blast furnace by reducing iron ore. The reducing agents are hydrogen, carbon monoxide and carbon. The reducing gas is produced by combustion of coke by blast air at about 1000^{o}C and decomposition of steam coming with blast air. The reactions are exothermic and produce large amount of heat in the lower part of the furnace. The reduction of iron ore by CO and H_2 is called indirect reduction and that by solid carbon is called direct reduction. If the reduction is more than normal by either of the process, it causes imbalance in the operation, as a result of which the temperature of the hearth increases. Since the solubility of silicon in the

molten metal increases with the temperature, silicon composition of the hot metal would increase. If the silica in the burden is increased, other conditions remaining uniform, silica content of slag will increase whereby corresponding increase inthe silicon content of molten metal will be noticed.

The two variables used for controlling hearth heat and hence the silicon content of hot metal are blast humidity and sinter-to-coke ratio. The effect of these two variables on silicon content are as follows:

Blast Humidity: It immediately reduces the temperature of the hearth due to decomposition of water vapour and correspondingly there will be a reduction in silicon content of the hot metal. However, increase in humidity increases the reducing power of the gases due to hydrogen andhence indirect reduction is favoured. Thus more carbon is available at the hearth for direct reduction which increases the temperature of the molten metal and hence its silicon content.

Sinter-to-Coke Ratio: When sinter-to-coke ratio is increased, the supply of coke per unit of sinter is decreased. The availability of carbon for direct reduction, in the lower region of blast furnace is reduced and hence hearth temperature is lowered. This will reduce the silicon content of hot metal. The physical properties of slag will also be affected. The long term supply of the reduced quantity of coke will depend upon the slag properties. The decrease in the coke may partly be compensated

by increasing the blast flow rate.

The input variables have long term and short term effects on the operation of a blast furnace. However, their effects are insignificant if the magnitude of variation is limited because of the high capacity of the system.

The data on the following input and output variables have been collected on Blast Furnace No.1 of capacity 2000 tons per day of Bokaro Steel Limited, Bokaro Steel City.

Input Variables:

(i) Sinter-to-coke ratio

(ii) Blast flow rate

(iii) Blast humidity

Output Variables:

(i) Hot metal temperature

(ii) Silicon content of hot metal

(iii) Sulphur content of hot metal

The metal was cast approximately 10 to 11 times per day with the cast period varying from two hours to three hours. Data have been collected for 350 consecutive casts comprising the operation of 36 days. The input variables were recorded continuously and sampled at one hour intervals. The time series for input variables were generated by averaging the values between successive casts. The observations on output variables were available only at the end of each cast period. Though the cast periods were not equal, for simplification in the

analysis, it has been assumed that all periods are equal and the cast period is considered as a 'time step'. The data have been reported in Appendix C and are shown in Figures 3.1 to 3.6. It should be noted that values of hot metal temperature were not available at all the casts. These points have not been included in Figure 3.4.

3.5.2 Preliminary Analysis of the Data:

All six variables, viz., sinter-to-coke ratio; blast humidity; blast flow rate; temperature, silicon content and sulphur content of hot metal have been named A,B,C,D,E and F respectively. It is seen from Figures 3.1 to 3.6 that there are two jumps in the series C and so it is nonstationary. Hence, it was divided into three parts, namely, C_1 , C_2 and C_3, each of which was treated as stationary. All three parts have different means and standard deviations. All other series are apparently stationary. The means and standard deviations of the variables are given in Table 3.1.

Out of 350 casts, the data on hot metal temperature were available only for 270 casts. Hence, the model was fitted to hot metal temperature with a sample size of 270 and then using the fitted model, the missing values were estimated and used in parameter estimation. This procedure was repeated until there was no change in parameter estimates in two successive iterations. Since the order of magnitude of mean and standard deviation is different for all series, the series were

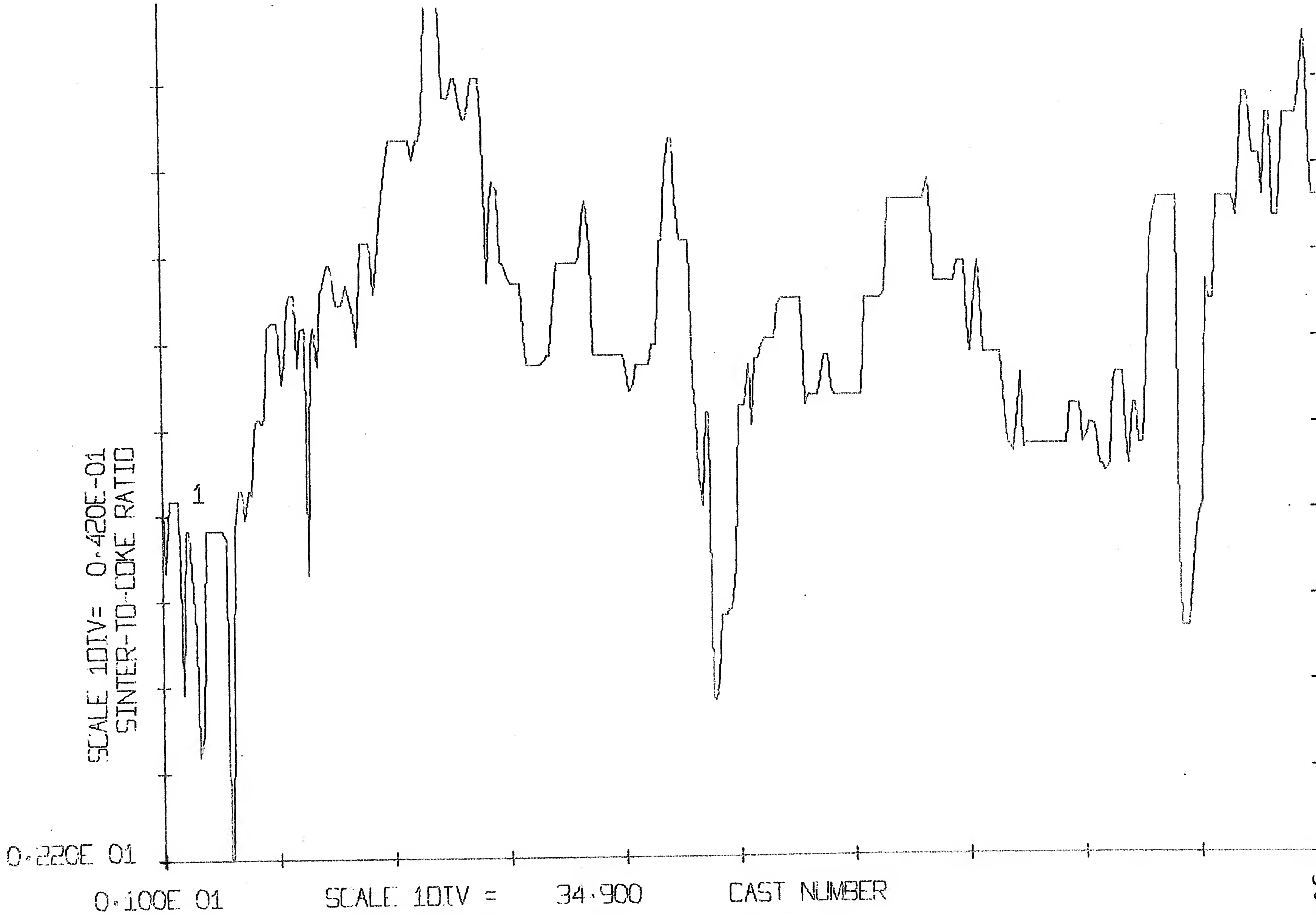


FIG. 3.1 SINTER-TO-COKE RATIO SERIES

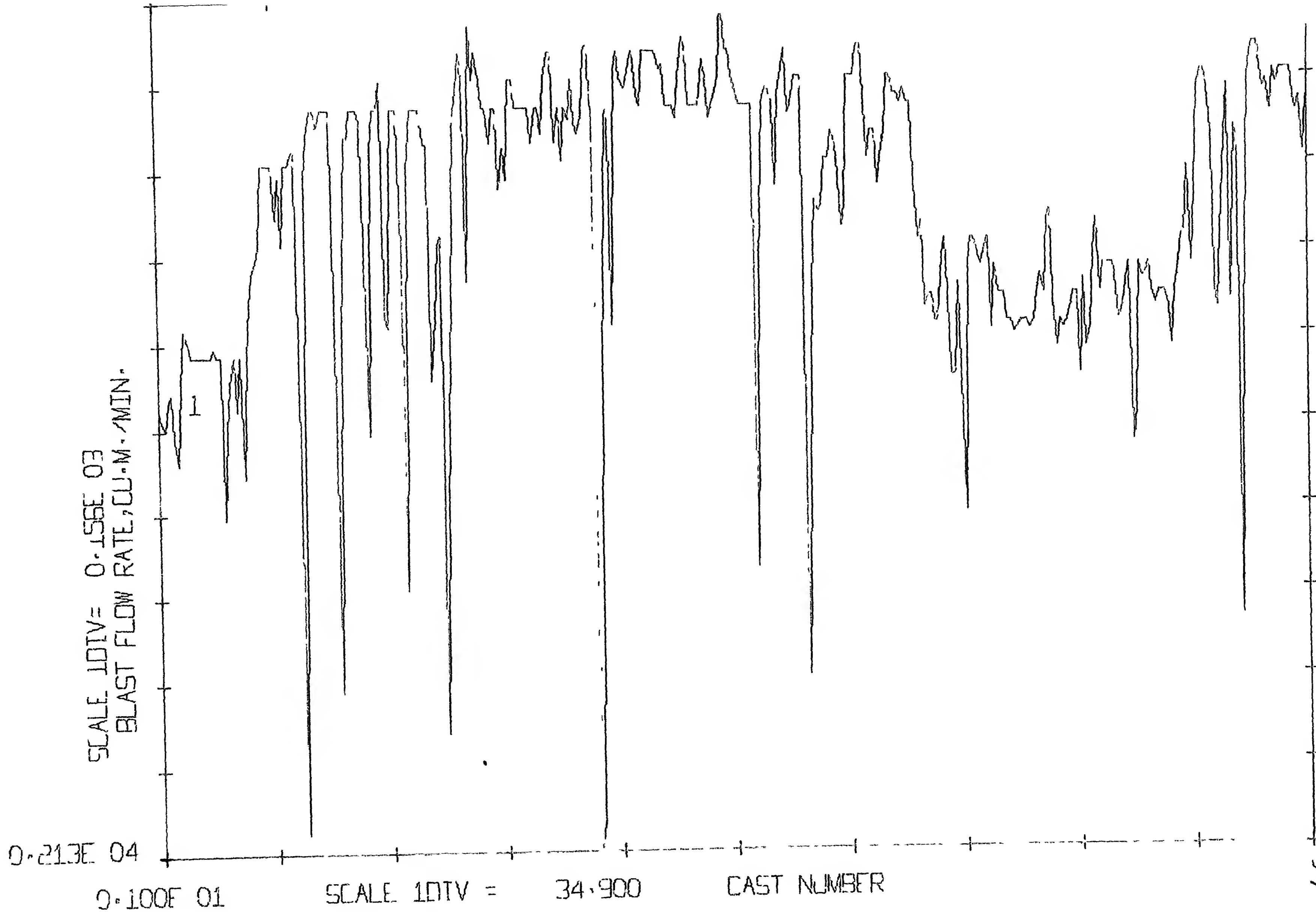


FIG 3.3 BLAST FLOW RATE SERIES

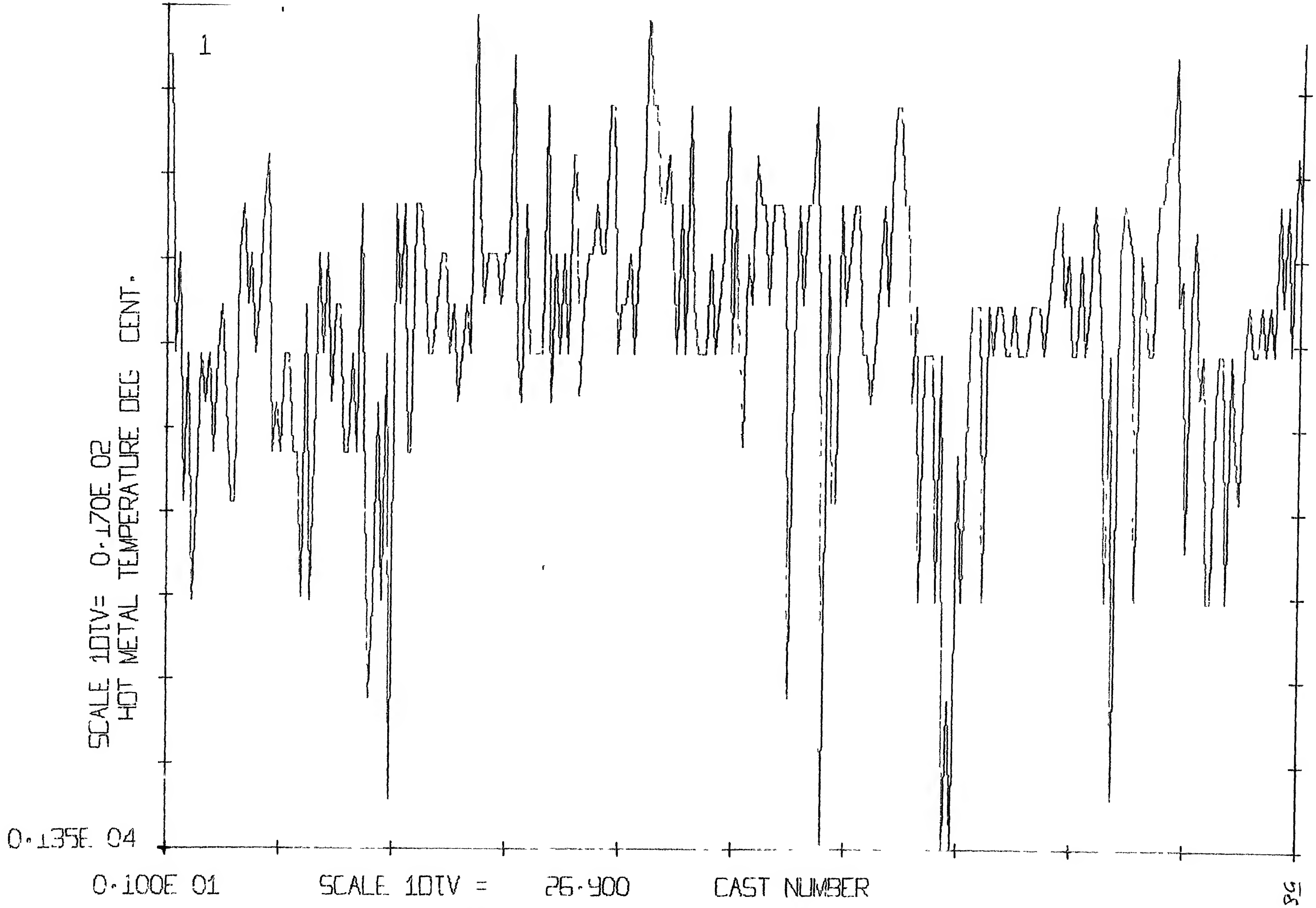


FIG. 3.4 HOT METAL TEMPERATURE SERIES

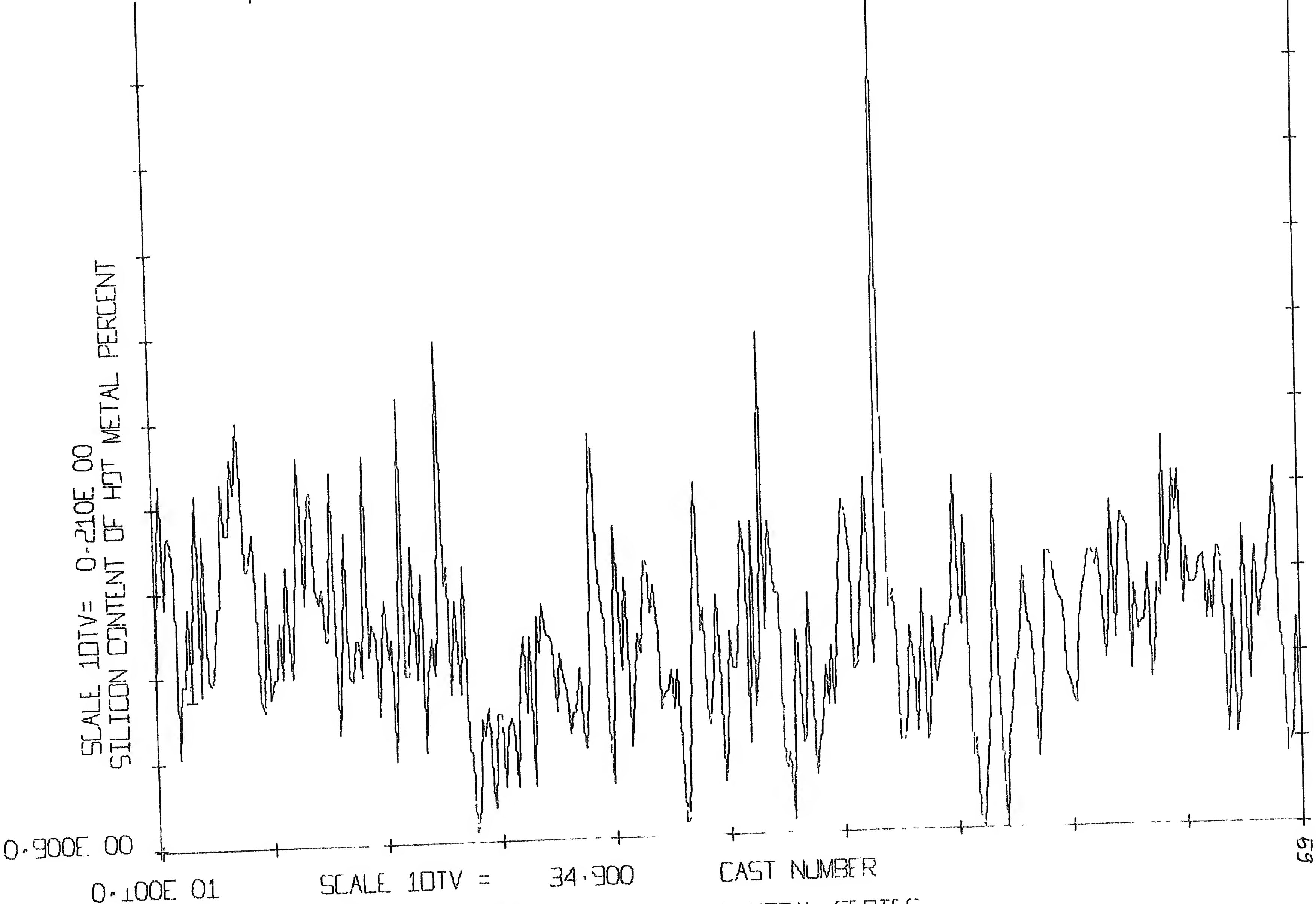


FIG. 3.5 SILICON CONTENT OF HOT METAL SERIES

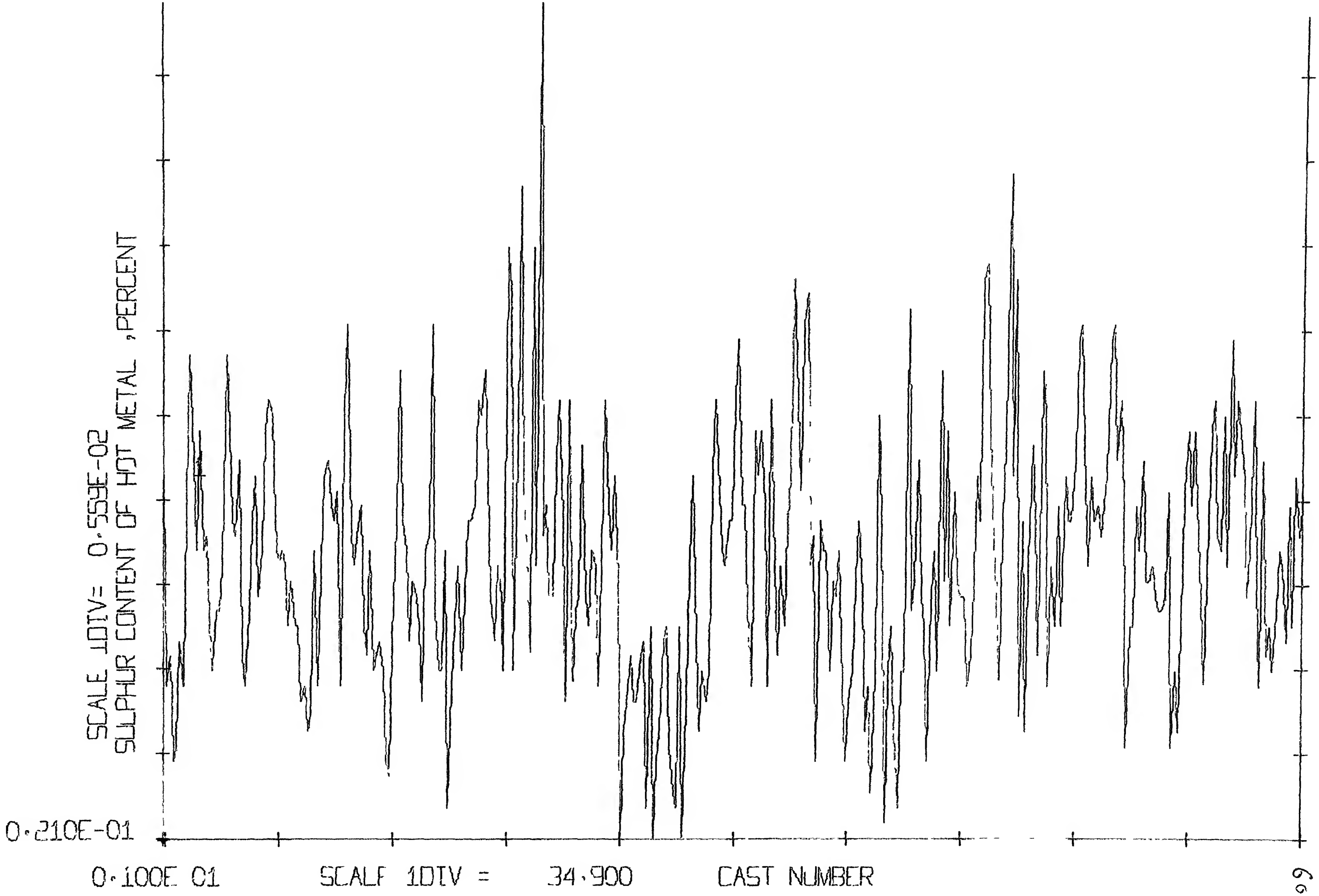


FIG. 3.6 SULPHUR CONTENT OF HOT METAL SERIES

TABLE 3.1

MEAN AND STANDARD DEVIATION OF DATA SERIES

Series	No. of observations	Mean	Standard deviation
Sinter-to-coke ratio, A	350	2.458	7.161×10^{-2}
Blast humidity, B	350	45.54, gm/m^3	3.841
Blast flow rate, C_1	43	3111.0, m^3/min	18.38
Blast flow rate, C_2	180	3411.0, m^3/min	27.53
Blast flow rate, C_3	127	3237.0, m^3/min	18.57
Hot metal temperature, D	270	1455.5, $^{\circ}C$	28.05
Silicon content of hot metal, E	350	1.404, per cent	0.0235
Sulphur content of hot metal, F	350	3.9×10^{-2}, per cent	8.29×10^{-3}

standardized using $\hat{\mu}$ the mean and $\hat{\sigma}$ the standard deviation of the raw series, viz.,

$$y(t) = [z(t) - \hat{\mu}]/\hat{\sigma}$$

so that all transformed series have zero mean and unit standard deviation. The standardized series are then used for further analysis.

3.5.3 Identification:

The aim of identification is to identify the presence of trend, persistence and periodic components in the time series.

Trend: The trend is identified by visual observation on plotting the raw data. If a trend is present, it can be estimated by regression analysis. A linear trend can be removed by taking the first difference of the series. The estimated trend component is subtracted from the series to give a trend free series which can be used for further processing.

Periodicity: The presence of cycles can be identified from the plot of the series and also from the autocorrelation function and spectral density function of the series. The presence of cycles is indicated by the occurrence of periodicity in the autocorrelation function and spikes at appropriate frequencies in the spectral density function. Once the frequencies are identified, the amplitudes and phase angles are estimated by harmonic coefficients or by regression analysis [69]. The cycles, after being identified, are subtracted from the trend

free series to give a series free from cyclicity.

<u>Persistence</u>: It refers to linkage between the values at a given time with earlier values and may be due to internal and/or external dependence. The trend free and cycle free series is analysed for the presence of persistence using autocorrelation function, partial autocorrelation function and spectral density function. The general behaviour of these will reveal whether the process is autoregressive (internal dependence), moving average (external dependence) or mixed. The autocorrelation function of an autoregressive process of order p tails off and its partial autocorrelation function has a cut-off after lag p. Conversely, the autocorrelation function of a moving average process of order q has a cut -off after lag q, while its autocorrelation function decays gradually. If both autocorrelation function and partial autocorrelation function tail off, a mixed process is indicated.

<u>Identification of the Plant Data</u>: The autocorrelation functions for all series were calculated usingEqn. 3.25. The partial autocorrelation functions and the standard errors were calculated usingEqn. 3.30 and Eqn. 3.31 respectively. The autocorrelation and partial autocorrelation functions, along with the 95 per cent confidence level for the latter have been plotted in Figures 3.7 to 3.12.

The raw spectral density function and smooth spectral dens function were calculated using Eqn. 3.32 and Eqn. 3.34. The

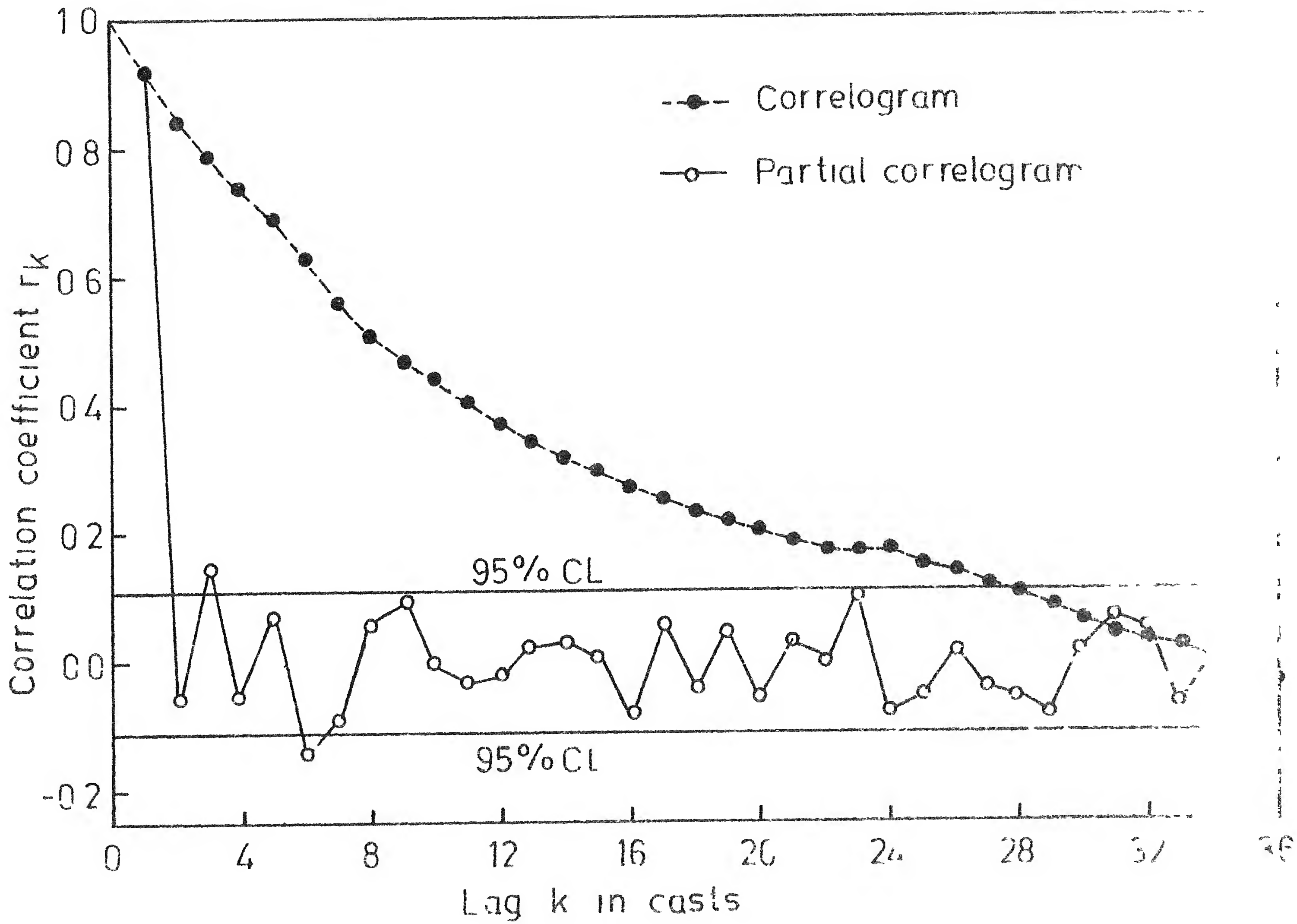


Fig 3 7 - Correlogram and partial correlogram of sinter-to-coke ratio series A

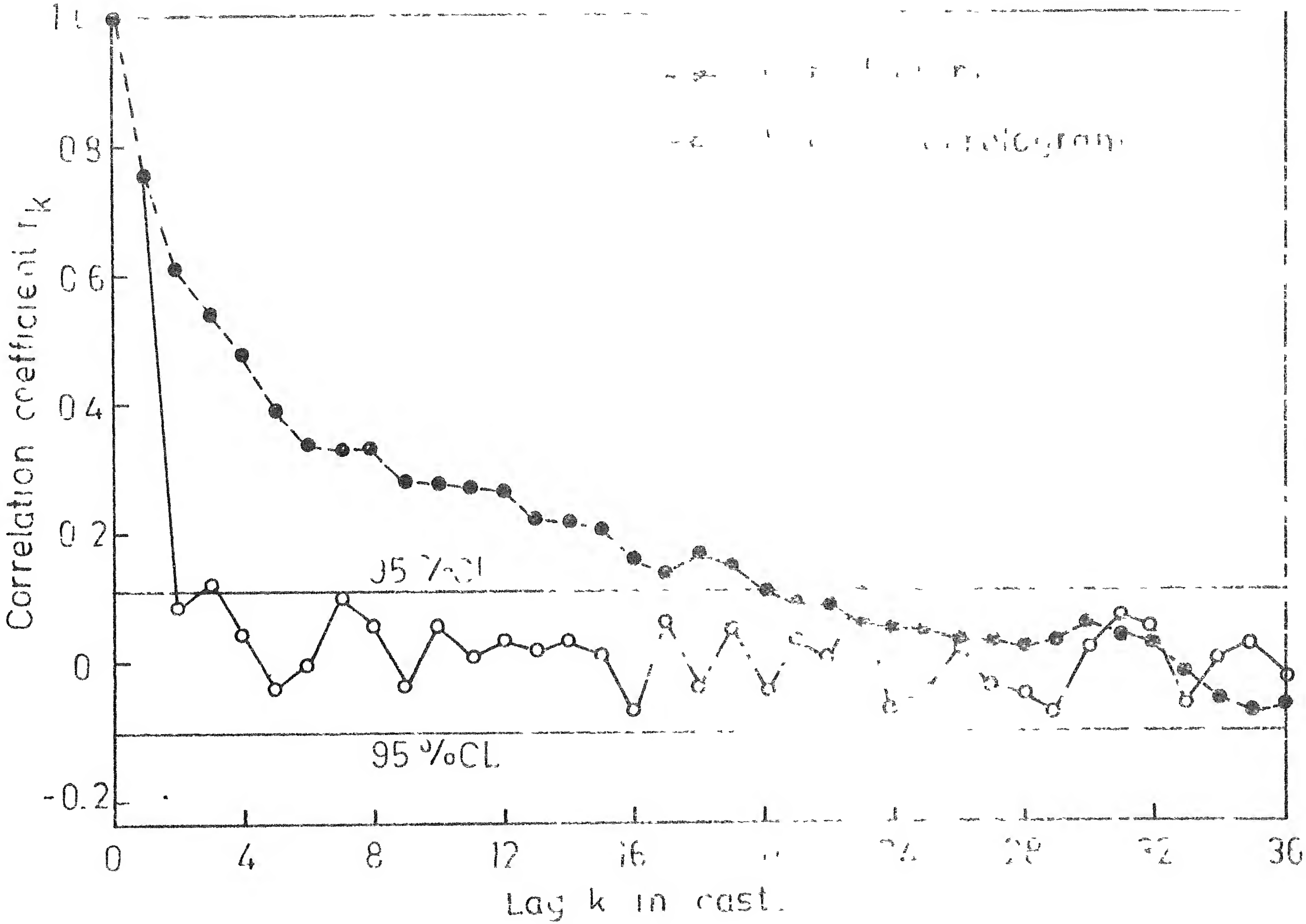


Fig 3.8 - Correlogram and partial correlogram of ... humidity series B

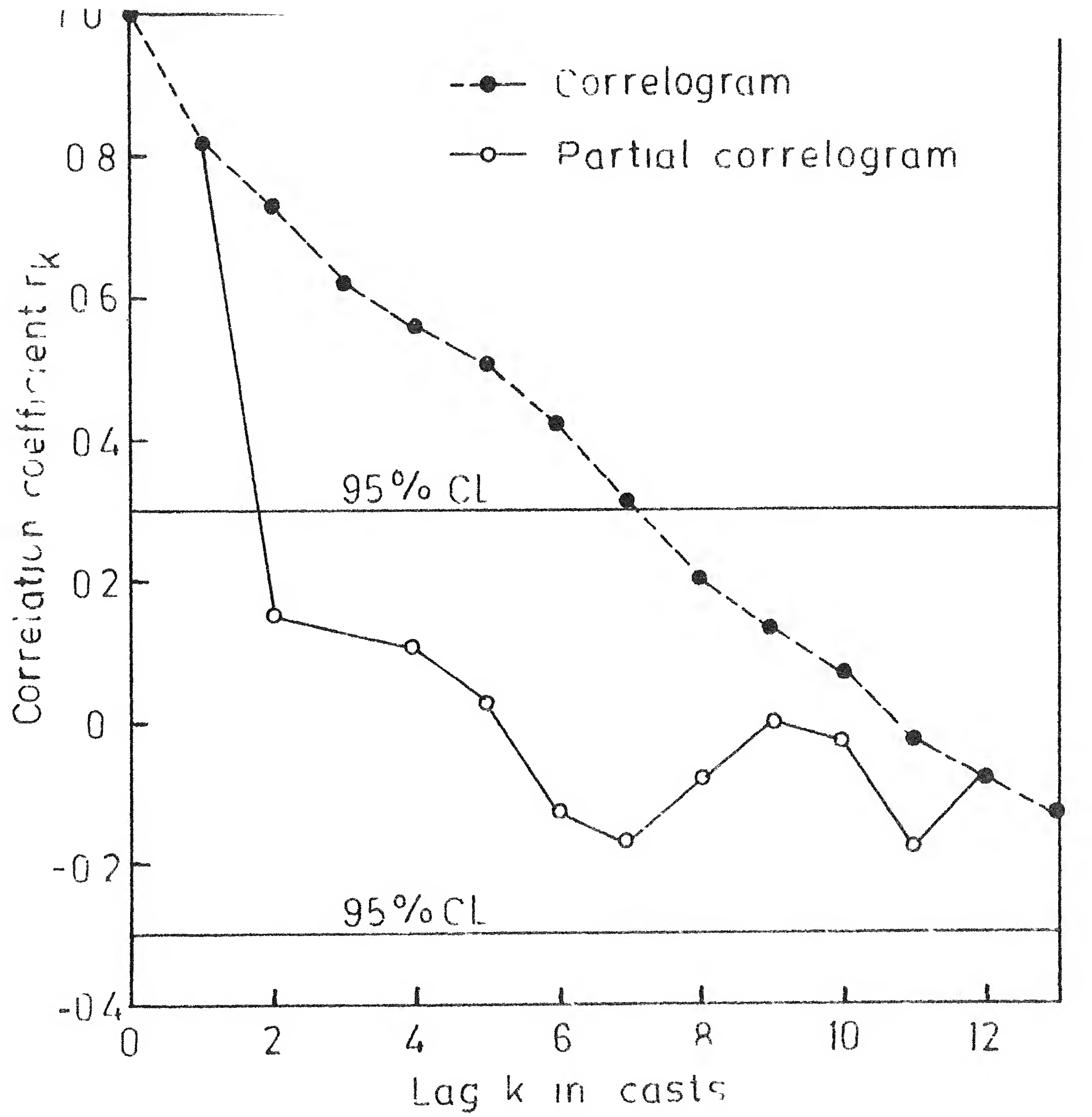


Fig 39a - Correlogram and partial correlogram of blast flow rate series C1

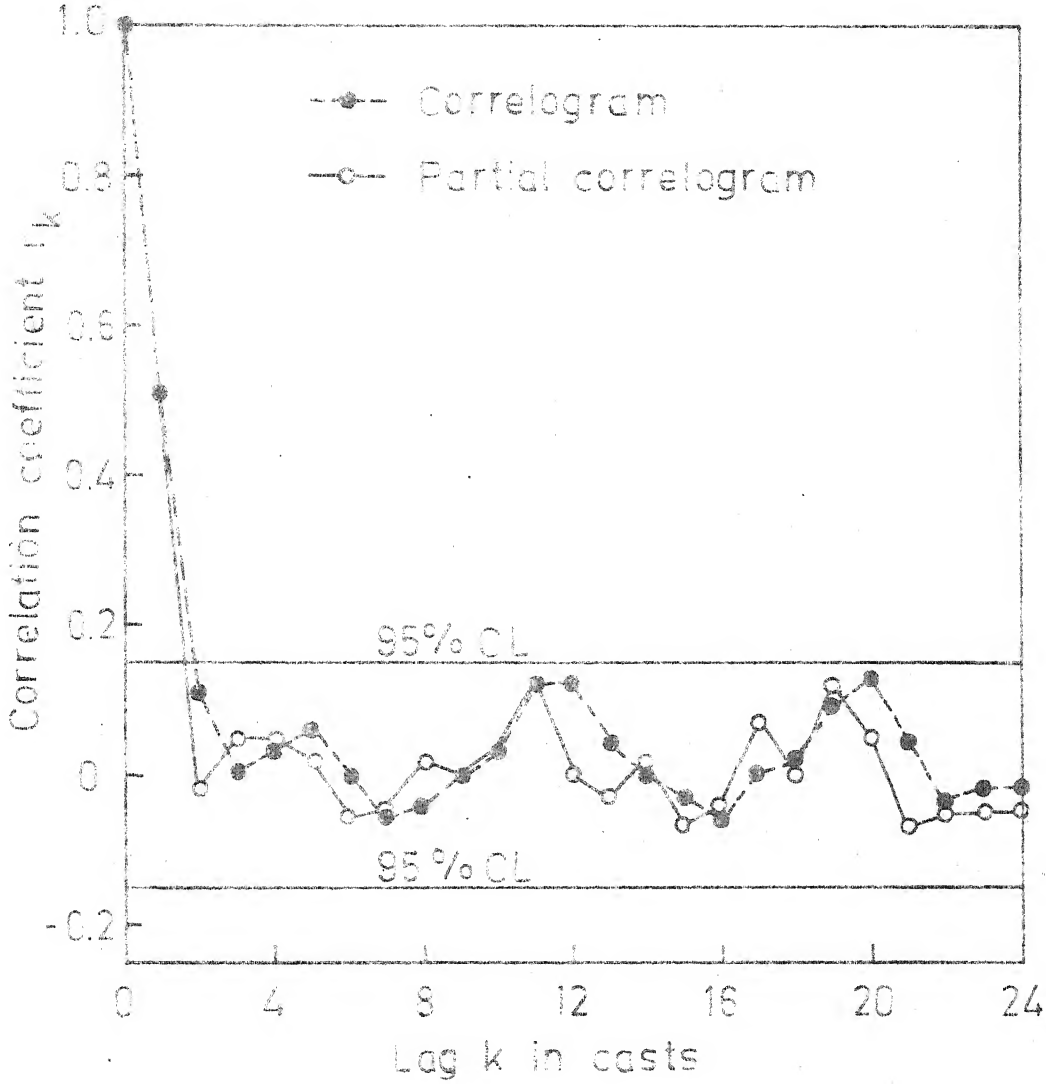


Fig. 3.9 b - Correlogram and partial correlogram of blast flow rate series C_2.

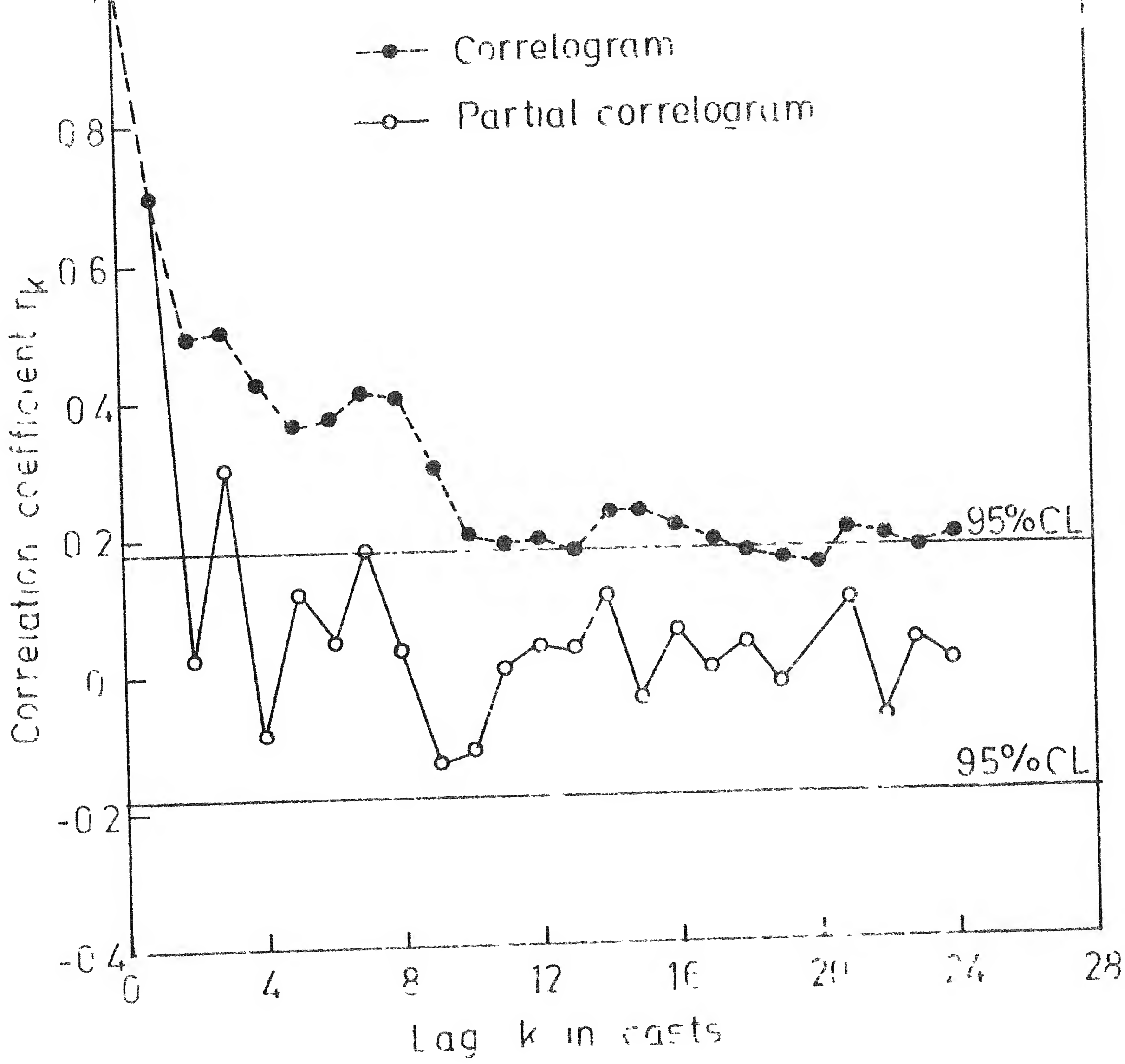


Fig 39c - Correlogram and partial correlogram of blast flow rate series F_3

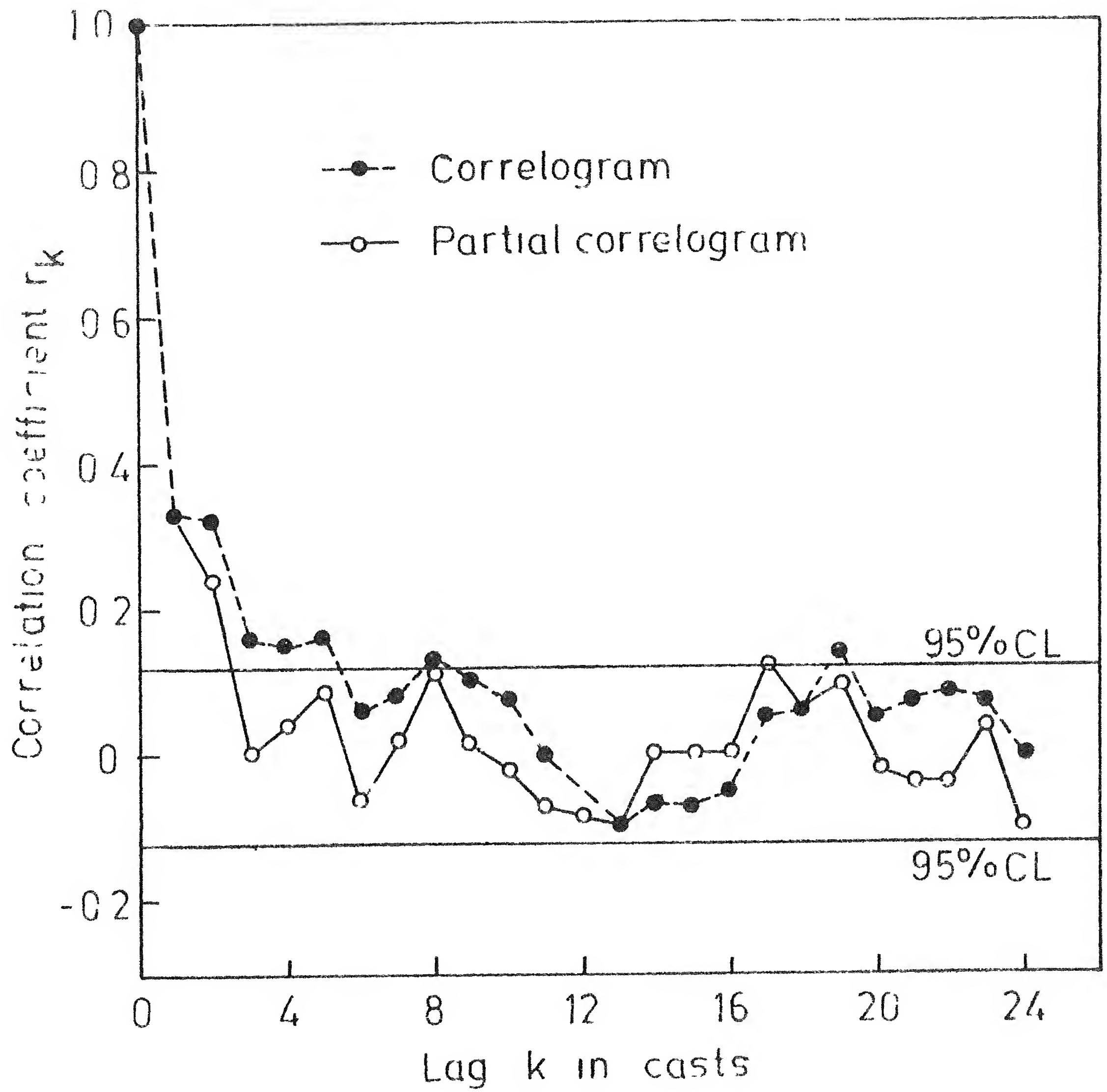


Fig 3 10 - Correlogram and partial correlogram of hot metal temperature series D.

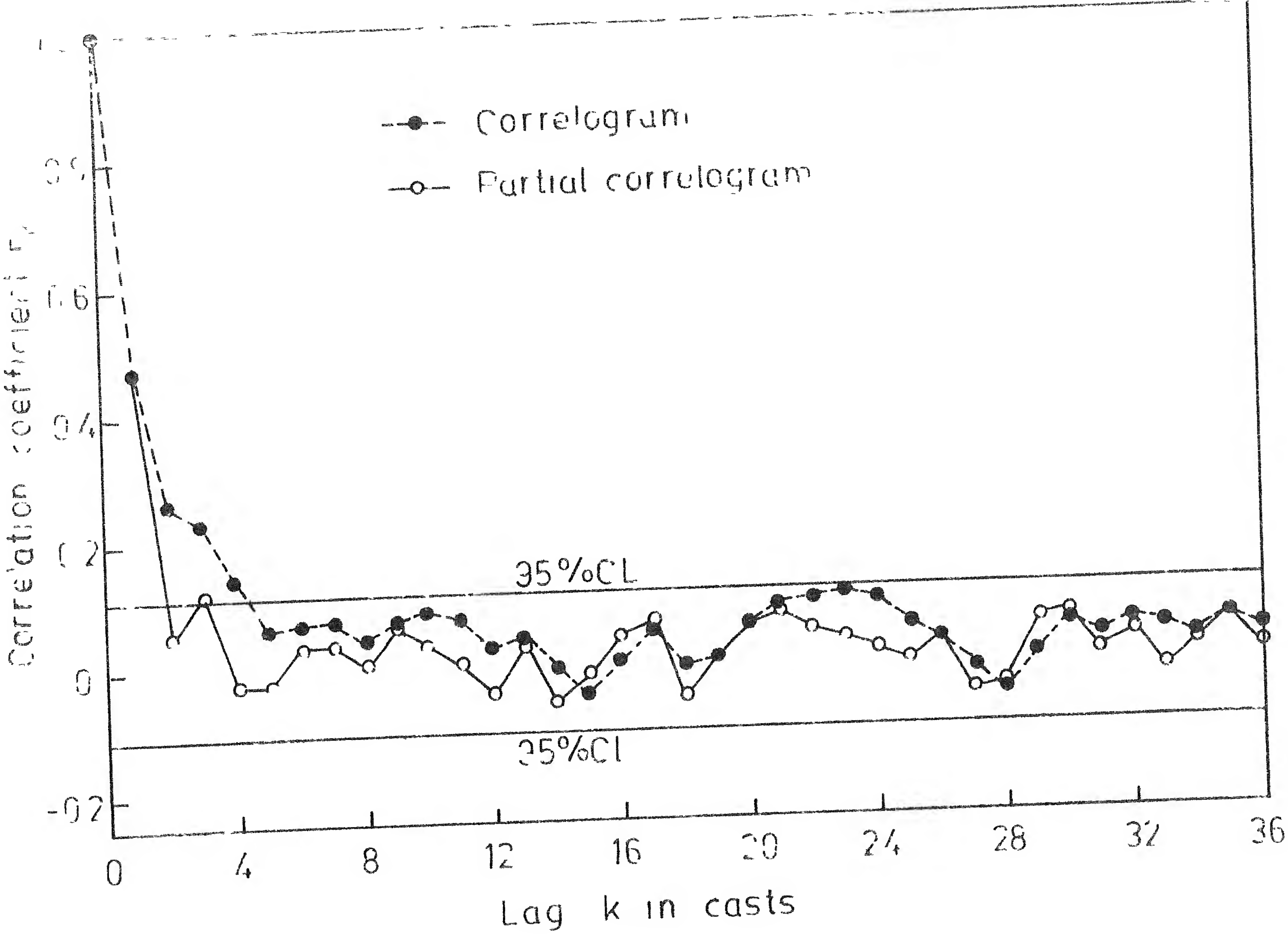


Fig 3.11 -Correlogram and partial correlogram of silicon content series F

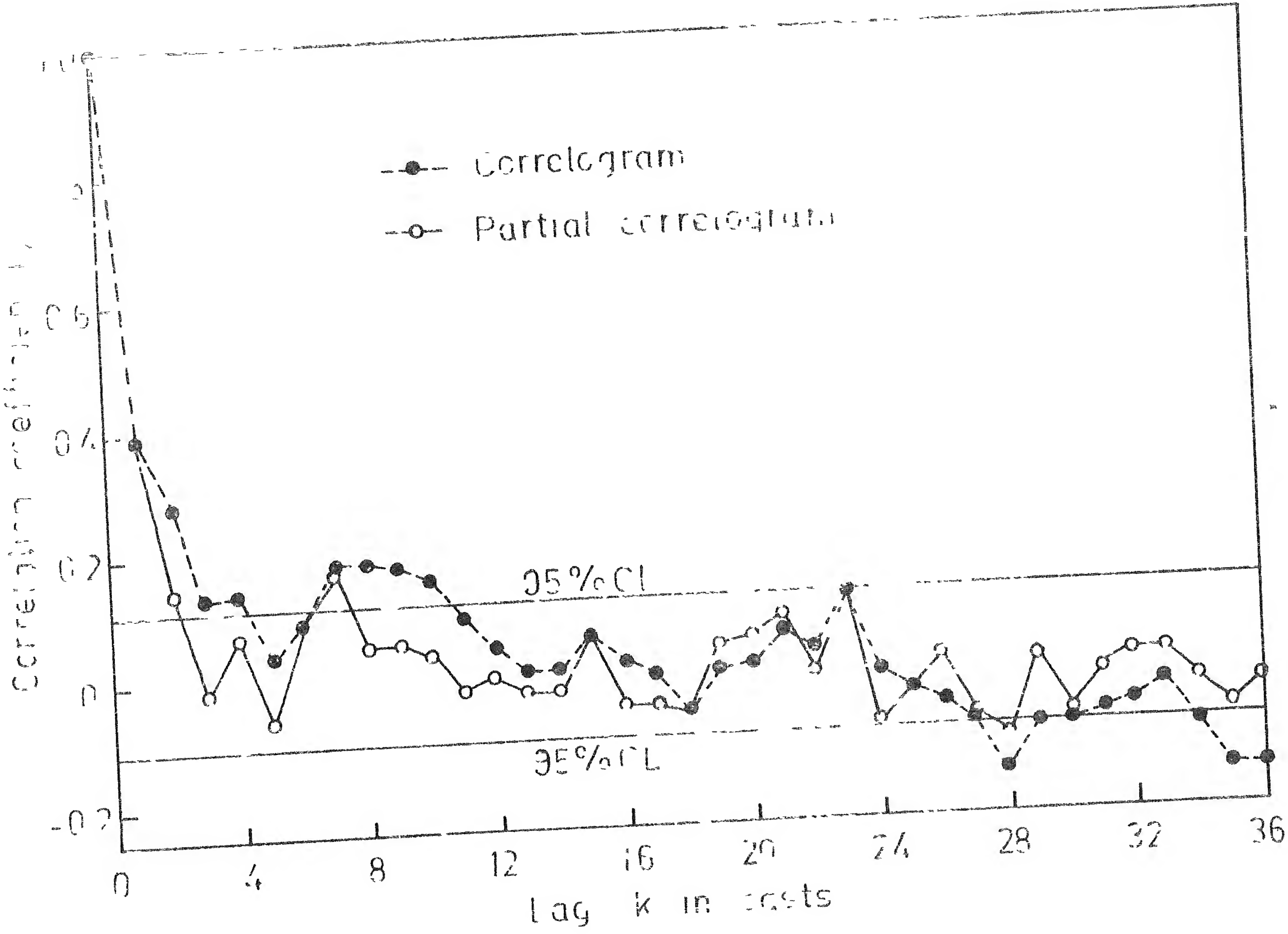


Fig 3.12 -Correlogram and partial correlogram of sulphur content series F

smooth spectra, for all series, have been plotted in Figures 3.13 to 3.18. They indicate no periodic components in any series. Based on the characteristics of autocorrelation function and partial autocorrelation function, a tentative identification of the models is given in Table 3.2. For most of the series more than one model was identified and the model with best fit was selected later at the stage of diagnostic checking.

3.5.4 <u>Estimation of Parameters:</u>

Having tentatively identified one or more models for a time series the next step is to obtain on the basis of some criterion, the best estimates of the parameters of the models. To start with, approximate estimates of the parameters are required. These are obtained from the autocorrelation coefficients of the process as follows:

<u>Autoregressive Process, AR(p)</u>: For an AR(p) process, the initial estimates are obtained by solving the Yule-Walker equations (see [65] and [66]).

$$\begin{bmatrix} r_1 \\ r_2 \\ \\ r_p \end{bmatrix} = \begin{bmatrix} 1 & r_1 & \cdots\cdots & r_{p-1} \\ r_1 & 1 & \cdots\cdots & r_{p-2} \\ \\ r_{p-1} & r_{p-2} & \cdots\cdots & 1 \end{bmatrix} \begin{bmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \\ \\ \hat{\varphi}_p \end{bmatrix} \qquad (3.35)$$

The initial estimate of the residual variance is obtained from

$$\hat{\sigma}_\varepsilon^2 = R_o \,[1 - \hat{\varphi}_1 r_1 - \hat{\varphi}_2 r_2 - \cdots\cdots - \hat{\varphi}_p r_p] \qquad (3.36)$$

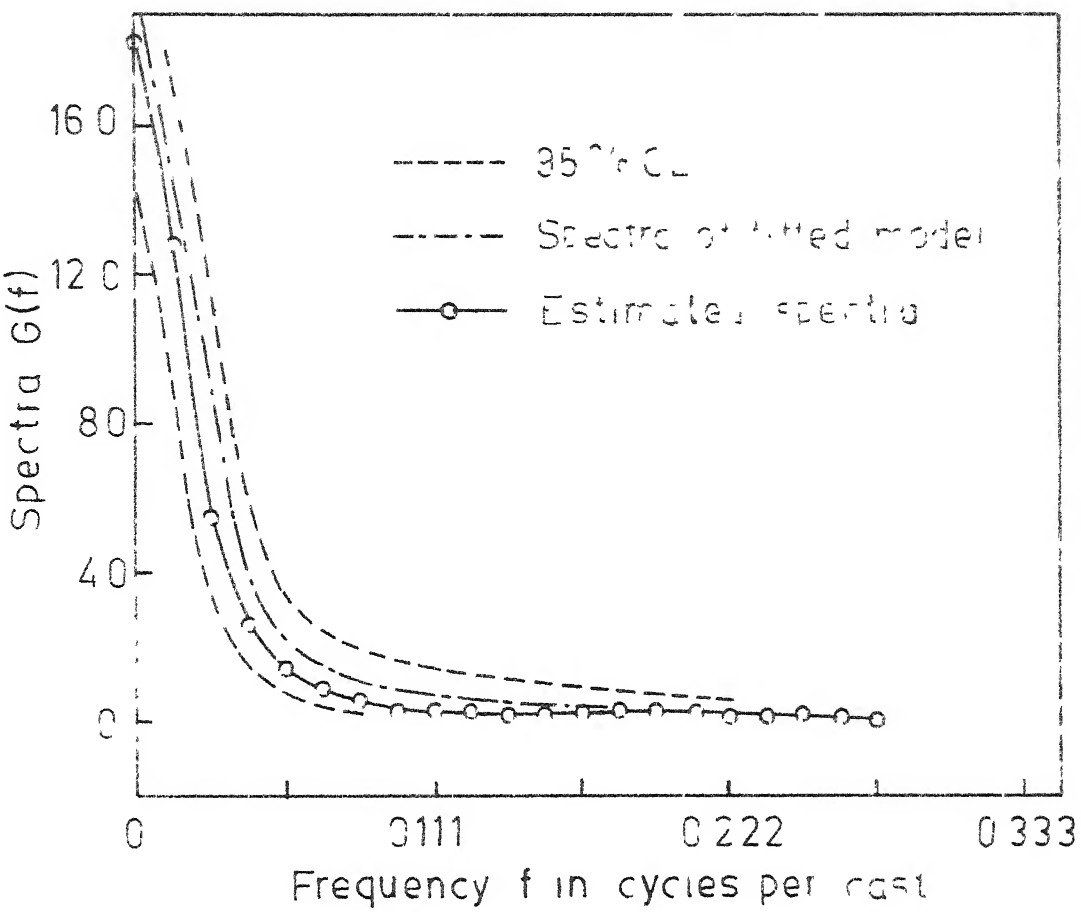


Fig [illegible] -Power spectra of sinter-to-coke ratio series A

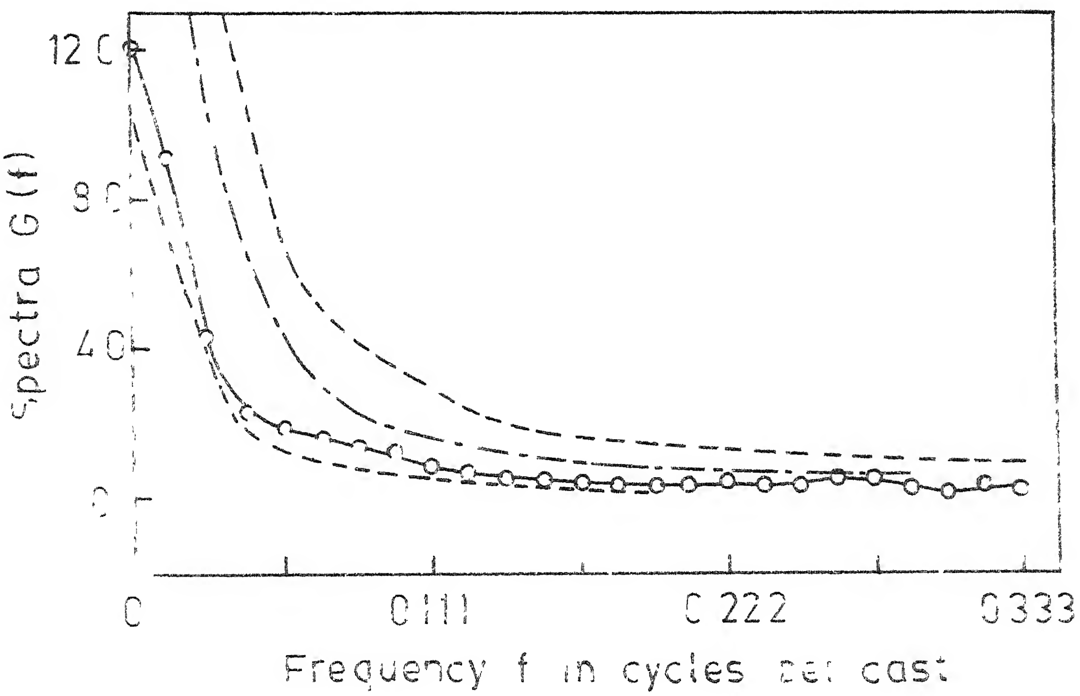
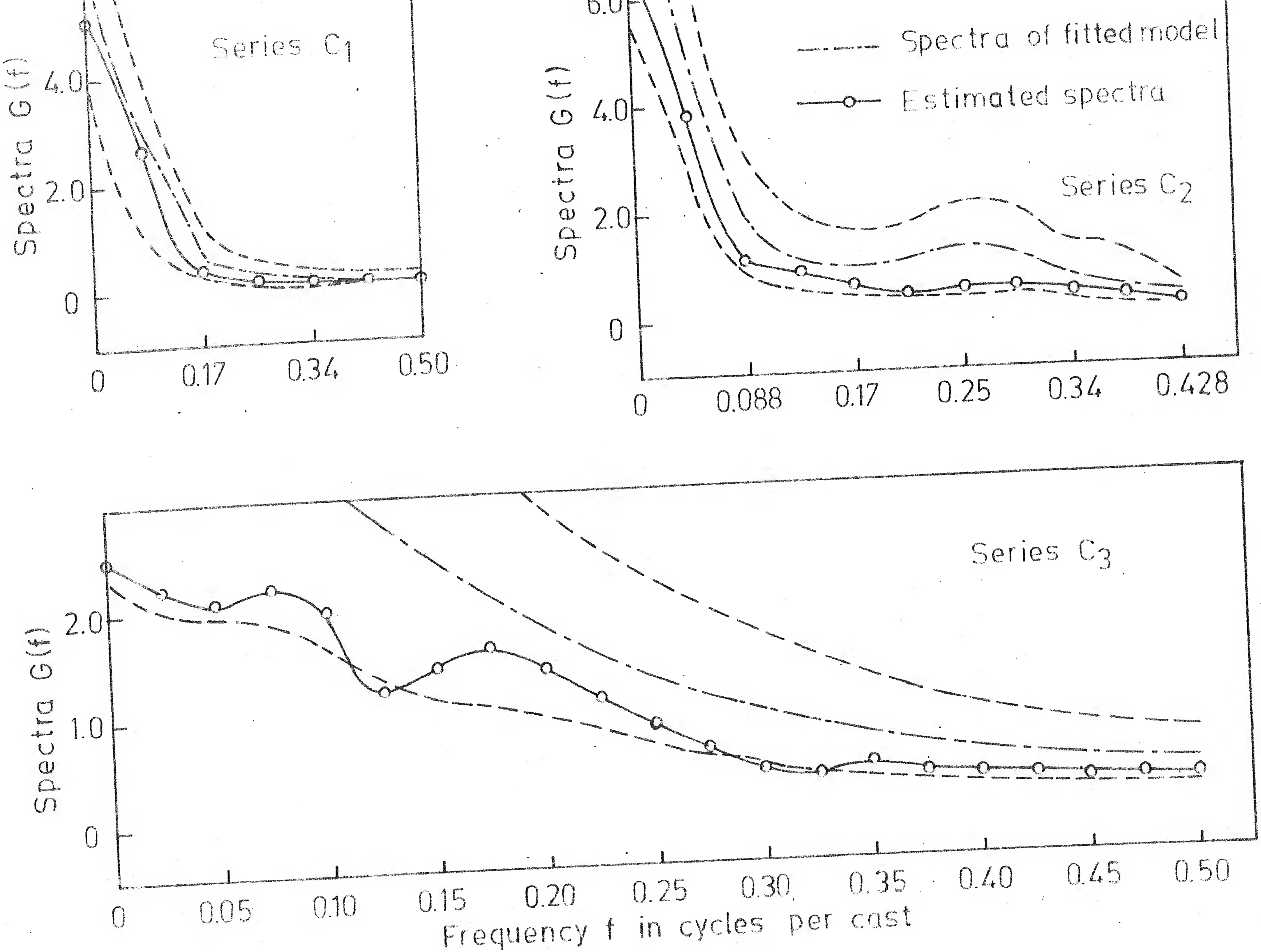


Fig [illegible] -Power spectra of blast humidity series B



Spectra of fitted model
Estimated spectra
Series C1
Series C2
Series C3
Spectra G(f)
Frequency f in cycles per cast

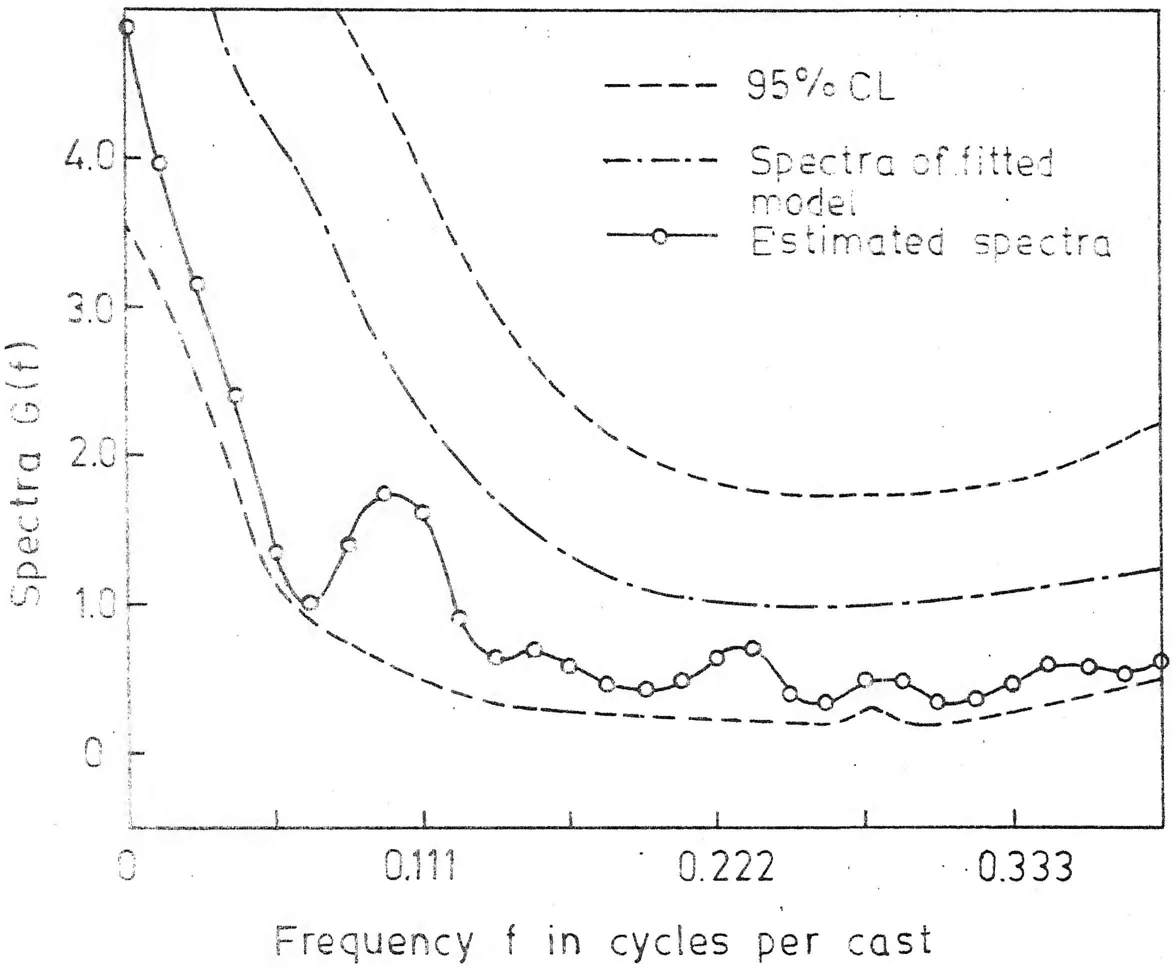


Fig. 3.16 - Power spectra of hot metal temperature series D.

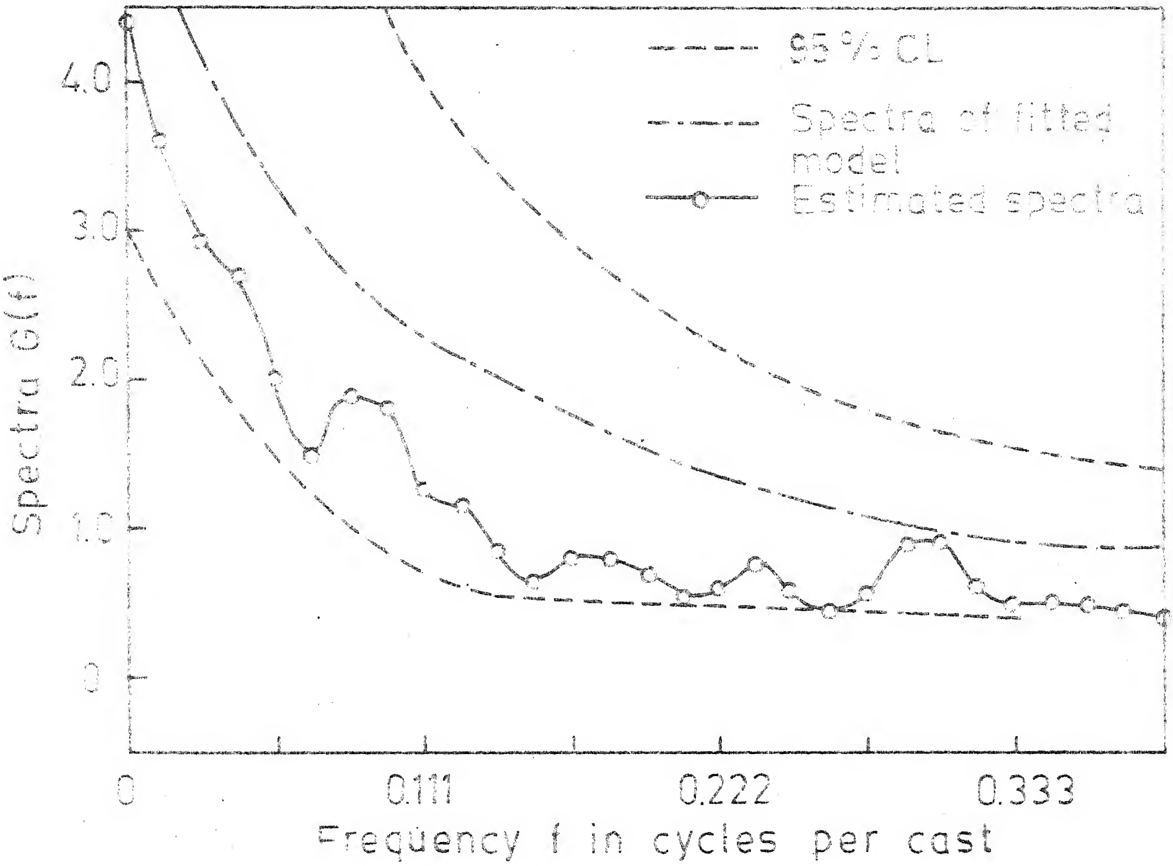


Fig.3.17 - Power spectra of silicon content series E.

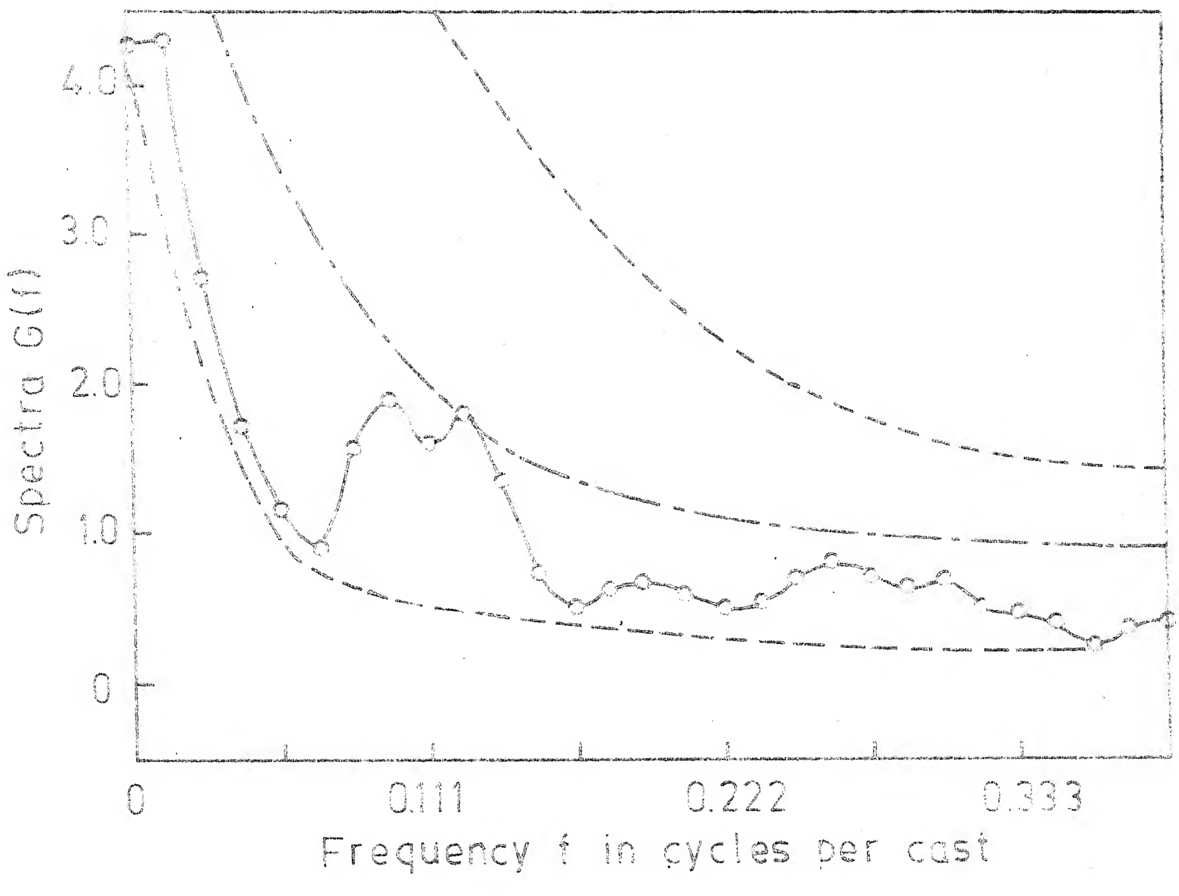


TABLE 3.2

TENTATIVE IDENTIFICATION OF MODELS FOR SERIES A TO F WITH INITIAL ESTIMATES

Series	No. of observations	Tentative identification	Identified Model	Residual variance, $\hat{\sigma}_\varepsilon^2$
A	350	(1,0,0)	$y(t) = 0.921\ y(t-1) + \varepsilon(t)$	0.150
		(2,0,0)	$y(t) = 0.958\ y(t-1) - 0.04\ y(t-2) + \varepsilon(t)$	0.149
		(3,0,0)	$y(t) = 0.986\ y(t-1) - 0.212\ y(t-2) + 0.154\ y(t-3) + \varepsilon(t)$	0.147
B	350	(1,0,0)	$y(t) = 0.761\ y(t-1) + \varepsilon(t)$	0.420
		(2,0,0)	$y(t) = 0.703\ y(t-1) + 0.076\ y(t-2) + \varepsilon(t)$	0.418
		(3,0,0)	$y(t) = 0.694\ y(t-1) + 0.009\ y(t-2) + 0.121\ y(t-3) + \varepsilon(t)$	0.413
C_1	43	(1,0,0)	$y(t) = 0.824\ y(t-1) + \varepsilon(t)$	0.323
C_2	180	(1,0,0)	$y(t) = 0.509\ y(t-1) + \varepsilon(t)$	0.741
		(0,0,1)	$y(t) = \varepsilon(t) + 0.695\ \varepsilon(t-1)$	0.570
		(1,0,1)	$y(t) = 0.211\ y(t-1) + \varepsilon(t) + 0.425\ \varepsilon(t-1)$	0.702
		(1,0,2)	$y(t) = -0.052\ y(t-1) + \varepsilon(t) + 0.675\ \varepsilon(t-1) + 0.189\ \varepsilon(t-2)$	0.707
C_3	127	(3,0,0)	$y(t) = 0.667\ y(t-1) - 0.182\ y(t-2) + 0.307\ y(t-3) + \varepsilon(t)$	0.472
D	270	(2,0,0)	$y(t) = 0.253\ y(t-1) + 0.239\ y(t-2) + \varepsilon(t)$	0.838
E	350	(1,0,0)	$y(t) = 0.471\ y(t-1) + \varepsilon(t)$	0.778
F	350	(1,0,0)	$y(t) = 0.396\ y(t-1) + \varepsilon(t)$	0.843
		(1,0,1)	$y(t) = 0.699\ y(t-1) + \varepsilon(t) - 0.368\ \varepsilon(t-1)$	0.833
		(2,0,1)	$y(t) = 0.202\ y(t-1) + 0.197\ y(t-2) + \varepsilon(t) + 0.140\ \varepsilon(t-1)$	0.826
		(1,0,2)	$y(t) = 0.484\ y(t-1) + \varepsilon(t) - 0.142\ \varepsilon(t-1) + 0.103\ \varepsilon(t-2)$	0.825

where, R_o is the variance of the original series.

Moving Average, MA(q) Process: For a MA(q) process the initia estimates for q unknowns $\hat{\theta}_1, \hat{\theta}_2, \ldots\ldots, \hat{\theta}_q$ are obtained from the following equations:

$$r_k = \frac{-\hat{\theta}_k + \hat{\theta}_1\hat{\theta}_{k+1} + \ldots + \hat{\theta}_{q-k}\hat{\theta}_q}{(1 + \hat{\theta}_1^2 + \hat{\theta}_2^2 + \ldots\ldots + \hat{\theta}_q^2)}, \quad k=1,2,\ldots q \qquad (3.37)$$

The initial estimate of the residual variance is obtained from

$$\hat{\sigma}_\varepsilon^2 = \frac{R_o}{(1 + \hat{\theta}_1^2 + \hat{\theta}_2^2 + \ldots + \hat{\theta}_1^2)} \qquad (3.38)$$

Mixed, ARMA(p,q) Process: In the general case, the calculation of initial estimates of an ARMA(p,q) process is based on the first (p+q+1) autocovariances $R_j[j=0,1,\ldots(p+q)]$ of stationar series z(t). The autoregressive parameters $\varphi_1, \varphi_2, \ldots, \varphi_p$ are estimated from the autocovariances $R_{q-p+1}, \ldots\ldots, R_{q+1}, R_{q+2}, \ldots$ $\ldots, R_{q+p}$ by solving the following set of equations:

$$\begin{aligned} R_{q+1} &= \hat{\varphi}_1 R_q + \hat{\varphi}_2 R_{q-1} + \ldots\ldots + \hat{\varphi}_p R_{q-p+1} \\ R_{q+2} &= \hat{\varphi}_1 R_{q+1} + \hat{\varphi}_2 R_q + \ldots\ldots + \hat{\varphi}_p R_{q-p+2} \qquad (3.39) \\ \\ R_{q+p} &= \hat{\varphi}_1 R_{q+p-1} + \hat{\varphi}_2 R_{q+p-2} + \ldots\ldots + \hat{\varphi}_p R_q \end{aligned}$$

Using the estimates $\hat{\underline{\varphi}}$ obtained from Eqns 3.39, the first (q+1) autocovariances R'_j, (j=0,1,....q) of the derived series

$$z'(t) = z(t) - \hat{\varphi}_1 z(t-1) - \ldots - \hat{\varphi}_p z(t-p)$$

are calculated from

$$R'_j = \begin{cases} \sum_{i=0}^{p} \sum_{k=0}^{p} \hat{\varphi}_{io}\, \hat{\varphi}_{ko}\, R_{|j+i-k|} & p > o \quad (\hat{\varphi}_{oo}=-1) \\ \\ R_j & p = o \end{cases} \tag{3.40}$$

where $j = 0,1,\ldots,q$ and $\hat{\varphi}_{io}$ stands for initial estimate of ith AR parameter. The initial estimates of the moving average parameters are obtained by a method suggested by Wilson [70] [65], using Newton-Raphson algorithm. Let

$$\underline{\tau} = (\tau_o, \tau_1, \ldots, \tau_q)^T$$

$$\text{where } \tau_o^2 = \sigma_\varepsilon^2; \quad \text{and } \theta_j = -\tau_j/\tau_o \qquad j = 1,2,\ldots,q \tag{3.41}$$

Then, if $\underline{\tau}^i$ is the estimate of $\underline{\tau}$ obtained at ith iteration, the new values at the (i+1) iteration are obtained from

$$\underline{\tau}^{i+1} = \underline{\tau}^i - \underline{\underline{T}}^{i^{-1}} \underline{f}'^i \tag{3.42}$$

where $\underline{f}' = (f'_o, f'_1, \ldots f'_q)^T$, $\quad f'_j = \sum_{i=0}^{q-j} \tau_i \tau_{i+j} - R'_j$

and

$$\underline{\underline{T}} = \begin{bmatrix} \tau_o & \tau_1 & \cdots\cdots & \tau_{q-1} & \tau_q \\ \tau_1 & \tau_2 & \cdots\cdots & \tau_q & 0 \\ \tau_2 & \tau_3 & \cdots\cdots & 0 & 0 \\ & & & & \\ \tau_k & 0 & & 0 & 0 \end{bmatrix} + \begin{bmatrix} \tau_o & \tau_1 & \tau_2 \cdots\cdots & \tau_q \\ 0 & \tau_o & \tau_1 \cdots\cdots & \tau_{q-1} \\ 0 & 0 & \tau_o \cdots\cdots & \tau_{q-2} \\ & & & \\ 0 & 0 & 0 & \tau_o \end{bmatrix} \tag{3.43}$$

The starting values are $\tau_o = \sqrt{R'_o}$, $\tau_1 = \tau_2 = \ldots = \tau_q = 0$

When $|f'_j| < \varepsilon$, $j = 0,1,\ldots,q$, for some prescribed values of ε, the process is considered to have converged and the parameter estimates are obtained from Eqn. 3.41.

Initial Estimates of the Identified Models: The initial estimates of the parameters were obtained by the methods discussed above. The form of the identified models with initial estimates and the residual variance are shown in Table 3.2.

Maximum Likelihood Estimation of Parameters: There are two methods for estimating the parameters of ARIMA models. They are

(i) Estimation using Bayes' theorem

(ii) Estimation using Maximum likelihood principle.

In the present study, the latter method suggested by Box and Jenkins [65] is used for estimation of the parameters. For a sample of N observations z(t), there will be a probability distribution $p(z/\underline{\xi}')$ depending upon some unknown parameters $\underline{\xi}'$. The vector $\underline{\xi}'$ refers to p+q+1 parameters ($\underline{\varphi}$, $\underline{\theta}$, σ_ε) of the ARIMA model. Before the data are available, $p(z/\underline{\xi}')$ will associate a density with each different outcome z(t), for fixed $\underline{\xi}'$. After the data have become available, there will be various values of $\underline{\xi}'$ which might have given rise to the fixed set of observations z(t). The appropriate function for this purpose is called the likelihood function $L(\underline{\xi}'/z)$, which is

of the same form as $p(z/\underline{\xi}')$, but in which $z(t)$ is now fixed and $\underline{\xi}'$ is variable. It is more convenient to work with the log-likelihood function $l(\underline{\xi}'/z) = \ln L(\underline{\xi}'/z)$.

The unconditional log-likelihood function corresponding to N observations assumed to be generated by an ARIMA model is given by

$$l(\underline{\varphi}, \underline{\theta}, \sigma_\varepsilon) = f(\underline{\varphi}, \underline{\theta}) - N \ln \sigma_\varepsilon - \frac{S(\underline{\varphi}, \underline{\theta})}{2\,\sigma_\varepsilon^2} \tag{3.44}$$

where $f(\underline{\varphi}, \underline{\theta})$ is a function of $\underline{\varphi}$ and $\underline{\theta}$. The unconditional sum of squares function $S(\underline{\varphi}, \underline{\theta})$ is given by

$$S(\underline{\varphi}, \underline{\theta}) = \sum_{t=-\infty}^{N} [\varepsilon(t) \,|\, \underline{\varphi}, \underline{\theta}, z]^2 \tag{3.45}$$

Usually $f(\underline{\varphi}, \underline{\theta})$ is important only for small N. For moderate and large values of N, Eqn. 3.44 is dominated by $S(\underline{\varphi}, \underline{\theta})/2\sigma_\varepsilon^2$ and thus contours of the unconditional sum of squares function in the space of the parameters ($\underline{\varphi}$, $\underline{\theta}$) are very nearly contours of log likelihood. Thus the least square estimates obtained by minimizing sum of squares given by Eqn.3.45, will provide very close approximations to the maximum likelihood estimates.

Calculation of unconditional sum of squares: Consider the general ARMA model

$$\varphi(B)\, \bar{x}(t) = \theta(B)\, \varepsilon(t) \tag{3.11}$$

The above equation can also be written in terms of backward model as

$$\varphi(F) \quad \bar{x}(t) = \theta(F)\ e(t) \qquad (3.46)$$

where F is the forward shift operator defined by

$$F\ \bar{x}(t) = \bar{x}\ (t+1) \text{ and } F^k\ \bar{x}(t) = \bar{x}(t+k) \qquad (3.47)$$

Taking conditional expectations of Eqns. 3.46 and 3.11, that is,

$$\varphi(F)[\bar{x}(t)] = \theta(F)[e(t)] \qquad (3.48)$$

$$\varphi(B)\ [\bar{x}(t)] = \theta(B)\ [\varepsilon(t)] \qquad (3.49)$$

Equation 3.48 is first used to compute back forecasts and then Eqn. 3.49 is used to generate the $[\varepsilon(t)]$'s. If the forecasts are found to be negligible in magnitude beyond some lead time Q', the recursive calculation goes forward with

$$\begin{aligned} [e(-j) \mid \underline{\varphi},\underline{\theta},z] &= 0 \qquad j = 0,1,2,\ldots \\ [\varepsilon(-j) \mid \underline{\varphi},\underline{\theta},z] &= 0 \qquad j > Q'-1 \end{aligned} \qquad (3.50)$$

The uncondition sum of squares is then calculated using

$$S(\underline{\varphi},\underline{\theta}) = \sum_{t=1-Q'}^{N} [\varepsilon(t)]^2 \qquad (3.51)$$

The least squares estimates of the parameters are obtained by minimizing the sum of squares given by Eqn. 3.51. For a purely autoregressive process residual $\varepsilon(t)$ is linear in the autoregressive parameters φ's; however, for a purely moving average process $\varepsilon(t)$ is nonlinear function of parameters. The linearization of the model is done using Taylor series. Let $\underline{\xi}$ represents p+q parameters $(\underline{\varphi},\ \underline{\theta})$. Expanding $[\varepsilon(t)]$ in a Taylor series about its value corresponding to the guessed set of parameter values $[\underline{\xi}_o]^T = \{\xi_{1,o}\ \ \xi_{2,o}, \ldots\ldots, \xi_{(p+q),o}\}$ one

gets

$$\varepsilon(t) = \varepsilon_o(t) - \sum_{i=1}^{p+q} (\xi_i - \xi_{i,o})\, x_{i,t} \tag{3.52}$$

where $\varepsilon_o(t) = [\varepsilon(t) \mid z, \underline{\xi}_o]$

and
$$x_{i,t} = - \left. \frac{\partial \varepsilon(t)}{\partial \xi_i} \right|_{\underline{\xi} = \underline{\xi}_o} \tag{3.53}$$

If $\underline{\underline{X}}'$ is the (N+Q)x (p+q) matrix of $x_{i,t}$, the (N+Q) equations (3.52) may be expressed as

$$\underline{\varepsilon}_o(t) = \underline{\underline{X}}' (\underline{\xi} - \underline{\xi}_o) + \underline{\varepsilon}(t) \tag{3.54}$$

where $\underline{\varepsilon}_o(t)$ and $\underline{\varepsilon}(t)$ are column vectors with (N+Q) elements. The adjustments $(\underline{\xi} - \underline{\xi}_o)$, which minimize $S(\underline{\xi}) = S(\underline{\varphi}, \underline{\theta}) = (\underline{\varepsilon}(t))^T (\underline{\varepsilon}(t))$ is then obtained by linear least squares. The values of parameters which minimize residual sum of squares are obtained by the constrained optimization method suggested by Marquardt [71, 72]. The computer algorithm for this method [73] is described in Appendix D. The residual variance is then estimated using

$$\hat{\sigma}^2_\varepsilon = \frac{S(\xi)}{N-p-q} \tag{3.55}$$

and the variance-covariance matrix by

$$\underline{\underline{V}}'(\underline{\xi}) = (\underline{\underline{X}}'^T \underline{\underline{X}})^{-1} \hat{\sigma}^2_\varepsilon \tag{3.56}$$

The standard error of the parameters is obtained by taking square root of the diagonal elements of the variance-covariance matrix. The 95 per cent confidence limit for ξ_i is given by $\pm$ 1.96 times the standard error of ξ_i.

The maximum likelihood estimates of the identified model were obtained by the methods discussed above. The summary of the models fitted to the series A to F is given in Table 3.3. Also shown in the table are the standard error of the parameters and the residual variance.

The analysis of multiple time series (Chapter 4), requires that the sample size of all variables should be equal. From Table 3.3 it can be seen that a model (2,0,0) has been fitted to 270 observations of hot metal temperature. The missing values were calculated using the equation

$$y(t) = \hat{\varphi}_1 y(t-1) + \hat{\varphi}_2 y(t-2) + (1-\hat{\varphi}_1-\hat{\varphi}_2)\hat{\mu}$$

and the parameters of the model were reestimated using 350 values. The process of calculating missing values and estimating the parameters was repeated until there was no change in the parameter estimates in two successive iterations. The final parameter estimates of the model for hot metal temperature are also given in Table 3.3.

3.5.5 Validation of the Model:

The model having been identified and parameters estimated, diagnostic checks are then applied to residuals to check the adequacy of the fitted model. The fitted model is considered to be adequate if the residual series constitute an independent normally distributed series. Normality of residuals is tested by Chisquare test and serial independence is tested by correlogram analysis and spectral analysis.

TABLE 3.3

SUMMARY OF MODELS FITTED TO SERIES A TO F

Series	No. of Observations	Identified Model	Fitted Models	Residual Variance
A	350	(1,0,0)	$y(t) = \underset{(\pm 0.019)}{0.923}\, y(t-1) + \varepsilon(t)$	0.148
		(2,0,0)	$y(t) = \underset{(\pm 0.053)}{0.970}\, y(t-1) - \underset{(\pm 0.053)}{0.049}\, y(t-2) + \varepsilon(t)$	0.147
		(3,0,0)	$y(t) = \underset{(\pm 0.053)}{0.977}\, y(t-1) - \underset{(\pm 0.073)}{0.197}\, y(t-2) + \underset{(\pm 0.052)}{0.151}\, y(t-3) + \varepsilon(t)$	0.145
B	350	(1,0,0)	$y(t) = \underset{(\pm 0.035)}{0.763}\, y(t-1) + \varepsilon(t)$	0.421
		(2,0,0)	$y(t) = \underset{(\pm 0.053)}{0.701}\, y(t-1) + \underset{(\pm 0.054)}{0.081}\, y(t-2) + \varepsilon(t)$	0.419
		(3,0,0)	$y(t) = \underset{(\pm 0.053)}{0.692}\, y(t-1) - \underset{(\pm 0.065)}{0.004}\, y(t-2) + \underset{(\pm 0.053)}{0.121}\, y(t-3) + \varepsilon(t)$	0.415
C_1	43	(1,0,0)	$y(t) = \underset{(\pm 0.071)}{0.897}\, y(t-1) + \varepsilon(t)$	0.263
C_2	180	(1,0,0)	$y(t) = \underset{(\pm 0.063)}{0.523}\, y(t-1) + \varepsilon(t)$	0.737
		(0,0,1)	$y(t) = \varepsilon(t) + \underset{(\pm 0.062)}{0.541}\, \varepsilon(t-1)$	0.729
		(1,0,1)	$y(t) = \underset{(\pm 0.122)}{0.282}\, y(t-1) + \varepsilon(t) + \underset{(\pm 0.118)}{0.335}\, \varepsilon(t-1)$	0.716
		(1,0,2)	$y(t) = \underset{(\pm 0.314)}{0.051}\, y(t-1) + \varepsilon(t) + \underset{(\pm 0.367)}{0.569}\, \varepsilon(t-1) + \underset{(\pm 0.198)}{0.153}\, \varepsilon(t-2)$	0.718

Table 3.3 (contd)

Series	No. of observations	Identified Model	Fitted Models	Residuals variance
C_3	127	(3,0,0)	$y(t) = \underset{(\pm 0.084)}{0.684}\, y(t-1) - \underset{(\pm 0.103)}{0.205}\, y(t-2) + \underset{(\pm 0.085)}{0.344} y(t-3) + \varepsilon(t)$	0.464
D	270	(2,0,0)	$y(t) = \underset{(\pm 0.051)}{0.257}\, y(t-1) + \underset{(\pm 0.051)}{0.217}\, y(t-2) + \varepsilon(t)$	0.839
	350	(2,0,0)	$y(t) = \underset{(\pm 0.051)}{0.237}\, y(t-1) + \underset{(\pm 0.050)}{0.308}\, y(t-2) + \varepsilon(t)$	0.804
E	350	(1,0,0)	$y(t) = \underset{(\pm 0.047)}{0.473}\, y(t-1) + \varepsilon(t)$	0.779
		(1,0,0)	$y(t) = \underset{(\pm 0.049)}{0.397}\, y(t-1) + \varepsilon(t)$	0.845
F	350	(1,0,1)	$y(t) = \underset{(\pm 0.095)}{0.657}\, y(t-1) + \varepsilon(t) - \underset{(\pm 0.118)}{0.514}\, \varepsilon(t-1)$	0.832
		(2,0,1)	$y(t) = \underset{(\pm 0.207)}{-0.222}\, y(t-1) + \underset{(\pm 0.076)}{0.354}\, y(t-2) + \varepsilon(t) + \underset{(\pm 0.217)}{0.582}\, \varepsilon(t-1)$	0.829
		(1,0,2)	$y(t) = \underset{(\pm 0.159)}{0.566}\, y(t-1) + \varepsilon(t) - \underset{(\pm 0.165)}{0.223}\, \varepsilon(t-1) + \underset{(\pm 0.082)}{0.0552}\, \varepsilon(t-2)$	0.834

Normality of Residuals: The sample space is divided into I mutually exclusive classes with a class frequency of 5 or more. Let p(i) be the probability that the variable belongs to ith class and if $\varepsilon(i)$ and $\varepsilon(i+1)$ are the limits of the ith class interval then

$$p(i) = F[\varepsilon(i+1)] - F[\varepsilon(i)] \tag{3.57}$$

The value of $F[\varepsilon(i)]$ is calculated from the Normal probability distribution table. Let f(i) be the observed frequency of the sample from the ith group. The Chi-square statistic is given by

$$\chi^2 = \sum_{i=1}^{I} \frac{[f(i) - Np(i)]^2}{Np(i)} \tag{3.58}$$

If J is the number of parameters estimated, then theoretically χ^2 has a chisquare distribution with (I-J-1) degrees of freedom. Let $\chi^2(\alpha')$ be the value of χ^2 at α' per cent confidence level for the above degrees of freedom as obtained from statistical tables. If the calculated value of χ^2 is less than the theoretical value, the residuals can be assumed to be normally distributed.

Correlogram Analysis: The autocorrelation function of a pure random process is uncorrelated and distributed approximately normally about zero with variance 1/N and has a standard error of $(1/N)^{\frac{1}{2}}$. The autocorrelation function of the residual series is plotted. If all autocorrelation coefficients lie within 95 per cent confidence limit ($\pm$ 1.96 times the standard

error) this indicates that the residual series is pure random and the model is considered to be adequate. However, it was pointed by Durbin (see [65]) that using $N^{-\frac{1}{2}}$ as standard error for residual autocorrelation function may underestimate the significance of apparent discrepencies. Hence, rather than considering the residual autocorrelation coefficients individually, it is desirable to consider the smallness of first K autocorrelations. This is done by calculating a Q-statistic given by

$$Q = N \sum_{i=1}^{K'} r_i^2(\hat{\varepsilon}) \tag{3.59}$$

The fitted model is considered to be adequate if Q is less than the value of $\chi^2(\alpha')$ at α' per cent confidence limit at $(K'-p-q)$ degrees of freedom.

<u>Spectral Analysis</u>: The test for independence of residuals can also be performed in the frequency domain. The spectral density function for a pure random (white noise) process, is constant over the frequency range $0 \leq f \leq f_c$ and is equal to variance of the process, which is same as the average of the computed values of spectra. The sample spectral density function is distributed about its population value according to a χ^2/ν distribution, where ν is the number of degrees of freedom given by

$$\nu = \frac{2N}{M} - \frac{2}{3} \tag{3.60}$$

where M stands for the number of lags estimated autocorrelation coefficients are used for spectral estimation. From the Chisquare Tables the values of $\chi^2(\alpha/2,\nu)$ and $\chi^2(1-\alpha/2,\nu')$ are read at a given significance level α. From these values χ^2/ν are determined. If G(f) denotes the spectral density for pure whitenoise and $G_k(f)$ that for given sample then $G_k(f)/G(f)$ has χ^2/ν' distribution and the series is not significantly different from pure random series at confidence level $(1-\alpha)$ if

$$\frac{\chi^2(1-\alpha/2,\nu)}{\nu} \leqslant \frac{G_k(f)}{G(f)} \leqslant \frac{\chi^2(\alpha/2,\nu')}{\nu'} \tag{3.61}$$

<u>Validation of the Models Fitted to the Plant Data:</u> The autocorrelation functions were calculated for residual series obtained from all models and the Q-statistic given by Eqn. 3.59 was calculated. The calculated and theoretical valuesoof chisquare have been given in Table 3.4. The simplest model for which the calculated value is less than the theoretical value at 95 per cent confidence level, was selected for each series.

The normality of the residuals was tested by chisquare test given by Eqn. 3.58. The calculated and theoretical value of chisquare at 95 per cent confidence level have been given in Table 3.5. It indicates that all residuals are nearly normally distributed.

TABLE 3.4

TEST FOR SERIAL INDEPENDENCE OF THE UNIVARIATE RESIDUALS

Series	Model	Q-Statistic: No. of degrees of freedom	Q-Statistic: Theoretical value at 95 per cent CL	Q-Statistic: Calculated value	Remarks
1	2	3	4	5	6
	(1,0,0)	11	19.70	25.30	Model rejected
		23	35.20	35.67	
		29	42.00	37.47	
A	(2,0,0)	10	18.30	24.32	Model rejected
		22	33.90	34.87	
		28	41.30	36.64	
	(3,0,0)	9	16.90	14.41	Model accepted
		21	32.70	25.42	
		27	40.10	27.25	
	(1,0,0)	11	19.70	20.42	Model rejected
		23	35.20	36.66	
		29	42.00	41.54	
B	(2,0,0)	10	18.30	18.52	Model rejected
		22	33.90	34.70	
		28	41.30	39.57	
	(3,0,0)	9	16.90	11.60	Model accepted
		21	32.70	22.55	
		27	40.10	27.79	
C_1	(1,0,0)	24	36.40	14.11	Model accepted
C_2	(1,0,0)	11	19.70	13.40	All models are adequate but(1,0,1) is selected because correlogram and partial-correlogram indicate that the model should have both AR and MA terms.
		23	35.20	19.60	
		29	42.60	20.61	
	(0,0,1)	11	19.70	8.58	
		23	35.20	13.64	
		29	42.60	15.02	
	(1,0,1)	10	18.30	5.01	
		22	33.90	10.49	
		28	41.30	11.65	
	(1,0,2)	9	16.90	4.60	
		21	32.70	10.34	
		27	40.10	11.43	

Table 3.4 (contd)

1	2	3	4	5	6
C_3	(3,0,0)	9 21 27	16.90 32.70 40.10	15.48 21.31 22.95	Model accepted
D	(2,0,0)	10 22 28	18.30 33.90 41.30	8.75 21.11 27.48	Model accepted
E	(1,0,0)	11 23 29	19.70 35.20 42.60	10.18 20.42 23.97	Model accepted
F	(1,0,0)	11 23 29	19.70 35.20 42.60	26.06 41.47 52.35	Model rejected
	(1,0,1)	10 22 28	18.30 33.90 41.33	17.54 32.65 42.20	Model rejected
	(1,0,2)	9 21 27	16.90 32.70 40.10	16.65 30.92 41.45	Model rejected
	(2,0,1)	9 21 27	16.90 32.70 40.10	15.78 27.67 37.75	Model accepted

TABLE 3.5

TEST FOR NORMALITY OF UNIVARIATE RESIDUALS

Series	Degrees of Freedom	Theoretical Value at 95 per cent CL	Calculated Value, Eqn. (3.58)
$\varepsilon_1(t)$	18	28.87	27.72
$\varepsilon_2(t)$	25	37.65	29.06
$\varepsilon_3(t)$	29	42.56	37.80
$\varepsilon_4(t)$	20	31.41	28.41
$\varepsilon_5(t)$	25	37.65	36.78
$\varepsilon_6(t)$	25	37.65	32.52

The residual autocorrelation functions for fitted models have been plotted in Figs. 3.19a to 3.19c. The power spectra for the fitted models have been plotted in Figures 3.20 to 3.25. Also shown in these figures are the 95 per cent confidence levels, calculated by Eqn. 3.26 for autocorrelation function and Eqn. 3.61 for spectral density function. In calculating the latter, the series were standardized to zero mean and unit standard deviation. Hence, the theoretical value of spectra which is equal to the variance of the series is 1.0. All these results support the fact that the residuals series are not different from the pure white noise series at 95 per cent confidence level.

Another test for adequacy of the fitted model is to calculate power spectra of the original series using the estimated AR and MA parameters. For a general ARMA (p,q) process, the power spectra is given by [74,75]

$$G(f) = 2\sigma_\varepsilon^2 \frac{\left|1-\theta_1 e^{-j2\pi f}-\theta_2 e^{-j4\pi f}-\ldots-\theta_q e^{-j2q\pi f}\right|^2}{\left|1-\varphi_1 e^{-j2\pi f}-\varphi_2 e^{-j4\pi f}-\ldots-\varphi_p e^{-j2p\pi f}\right|^2} \tag{3.62}$$

The above equation can be simplified for different values of p and q. Power spectra for some processes are as follows:

Second order AR process:

$$G(f)=\frac{2\sigma_\varepsilon^2}{[1+\varphi_1^2+\varphi_2^2 - 2\varphi_1(1-\varphi_2)\mathrm{Cos}2\pi f - 2\varphi_2\mathrm{Cos}4\pi f]} \tag{3.63}$$

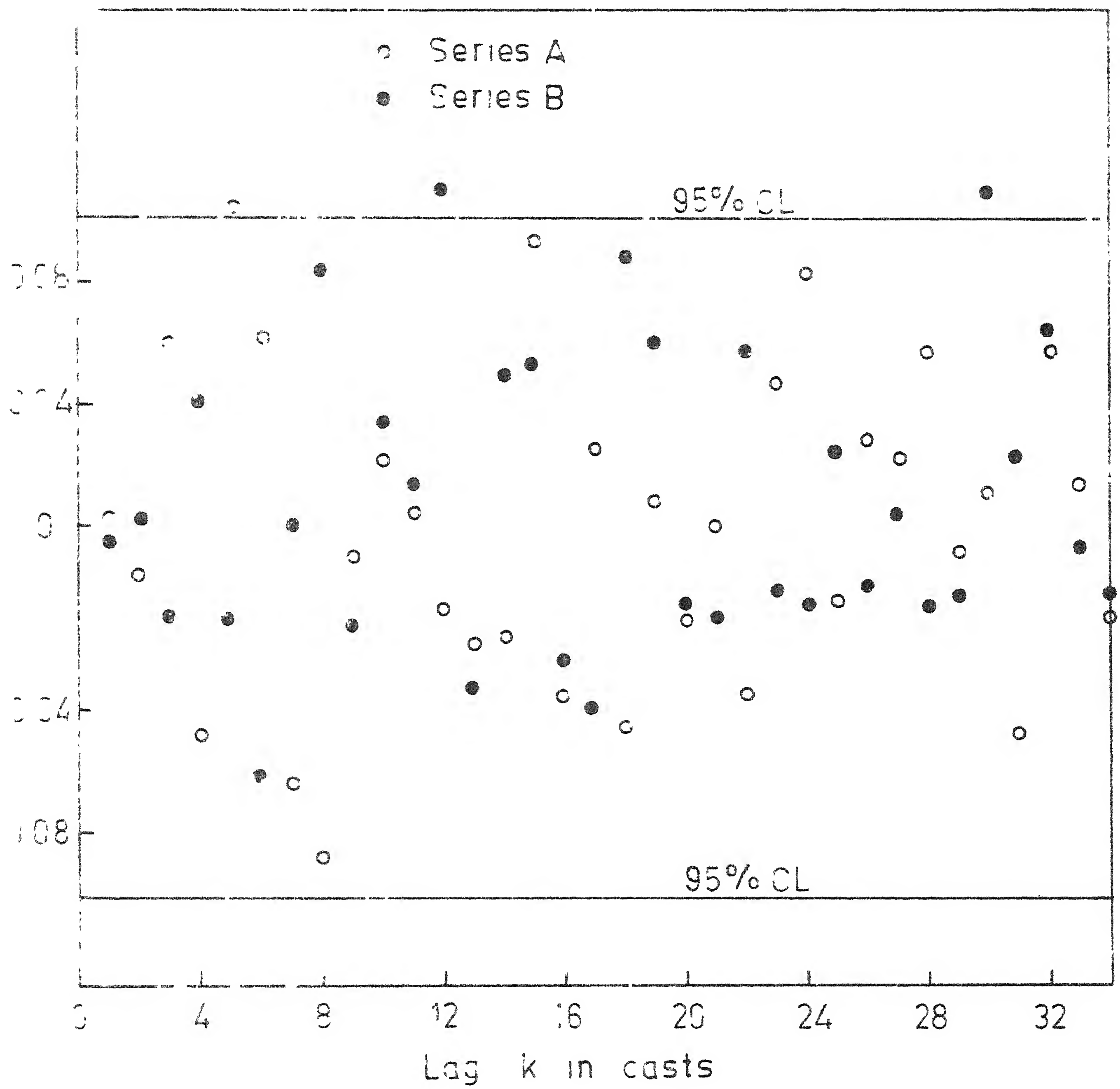


Fig 3 19a- Correlogram of univariate residuals A and B

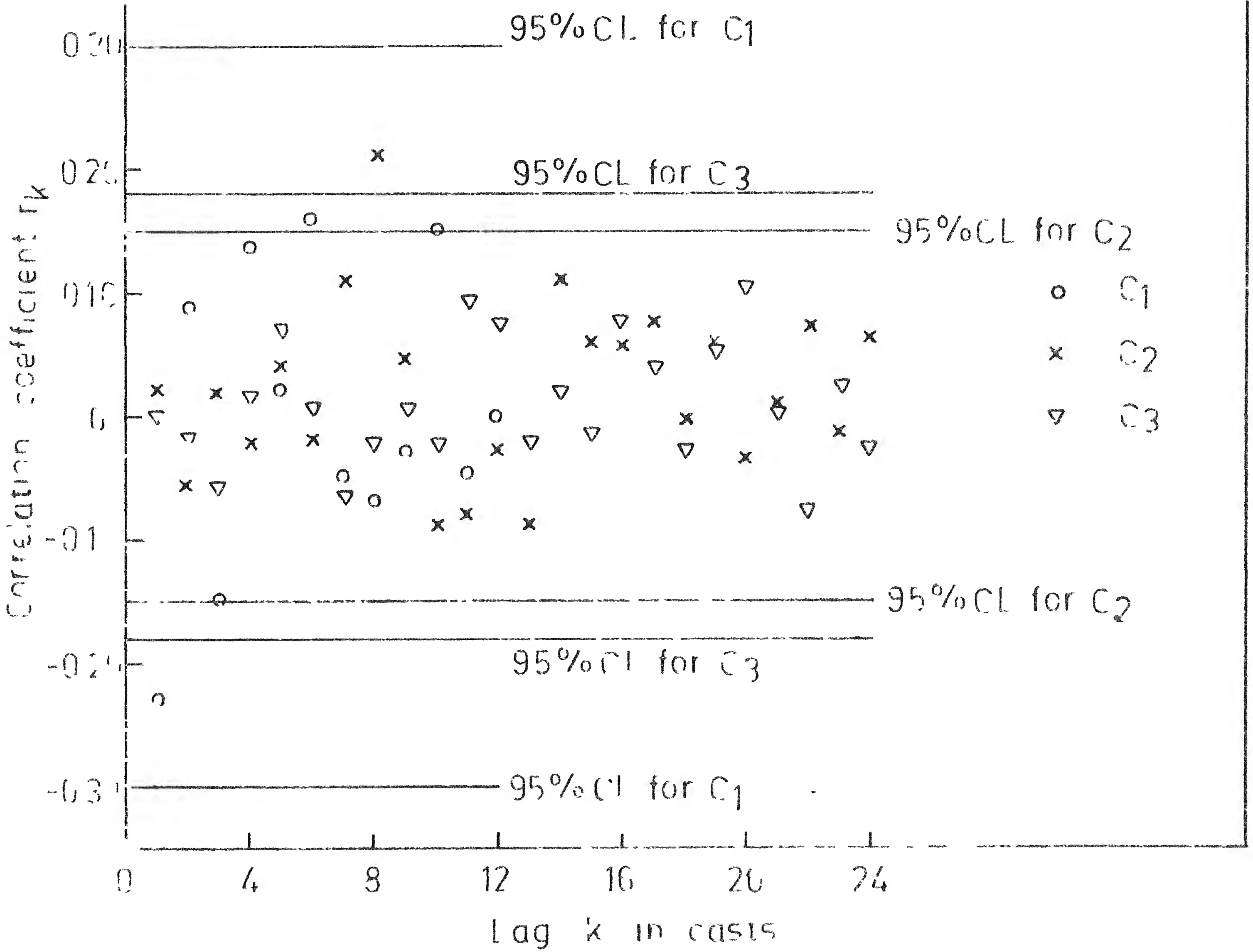


Fig 5.10 - Correlogram of univariate residuals C_1, C_2 and C_3

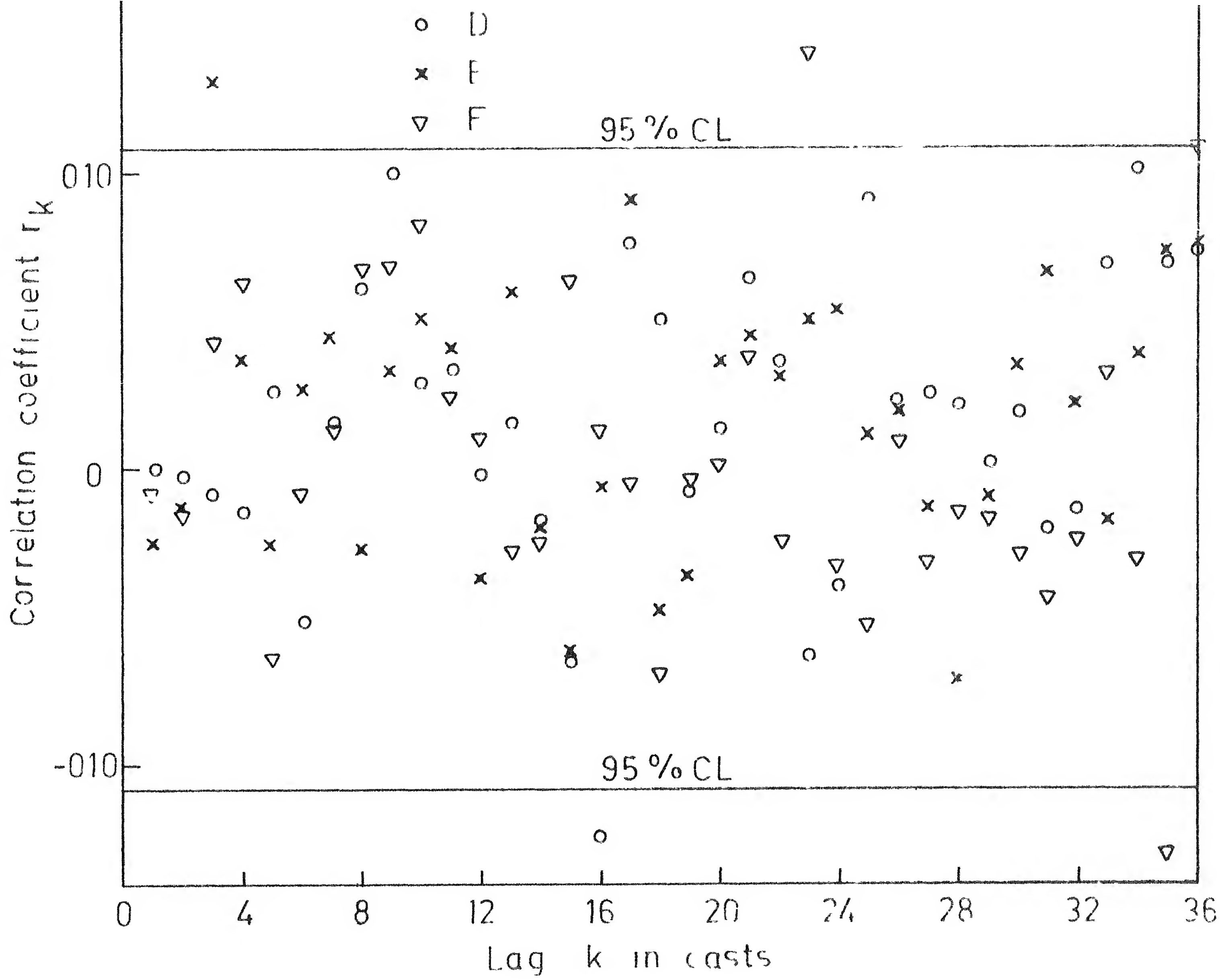
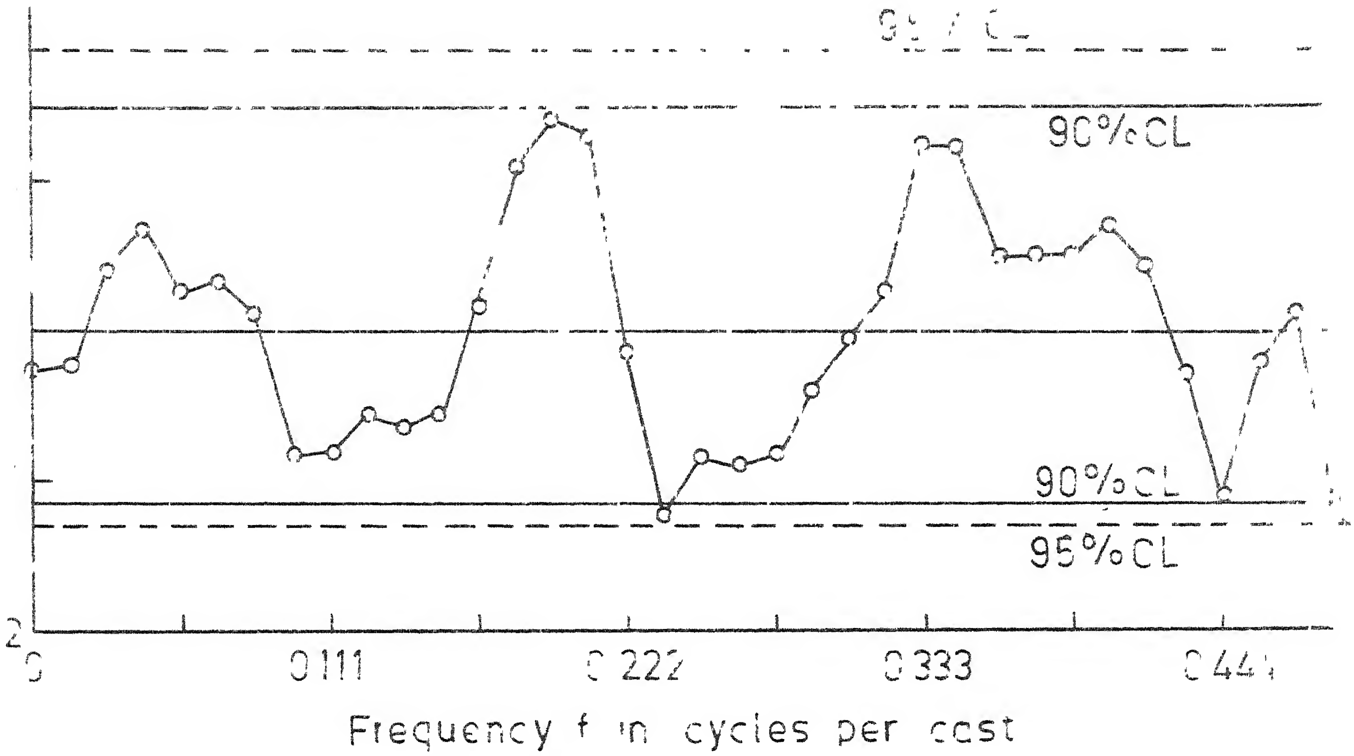
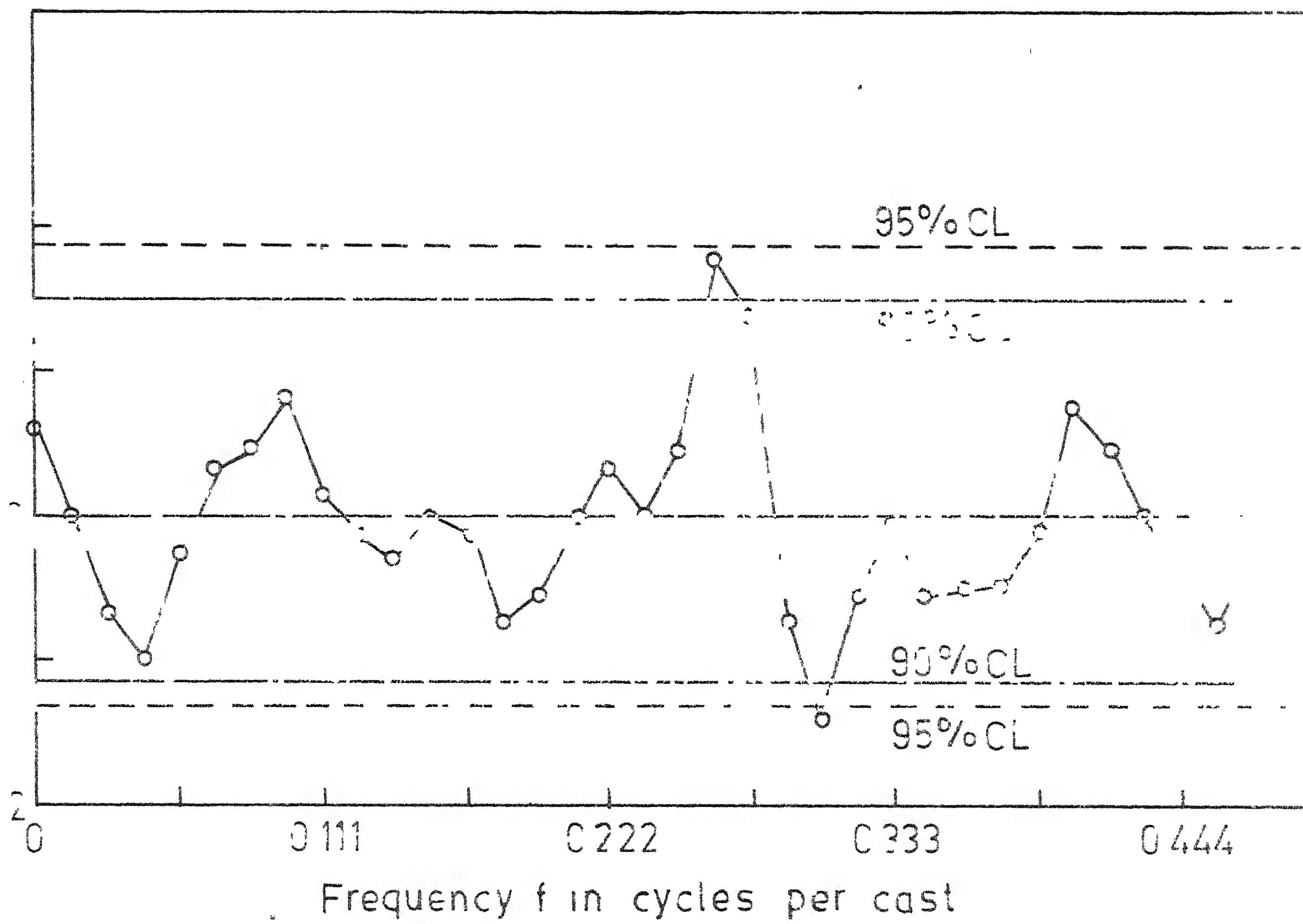


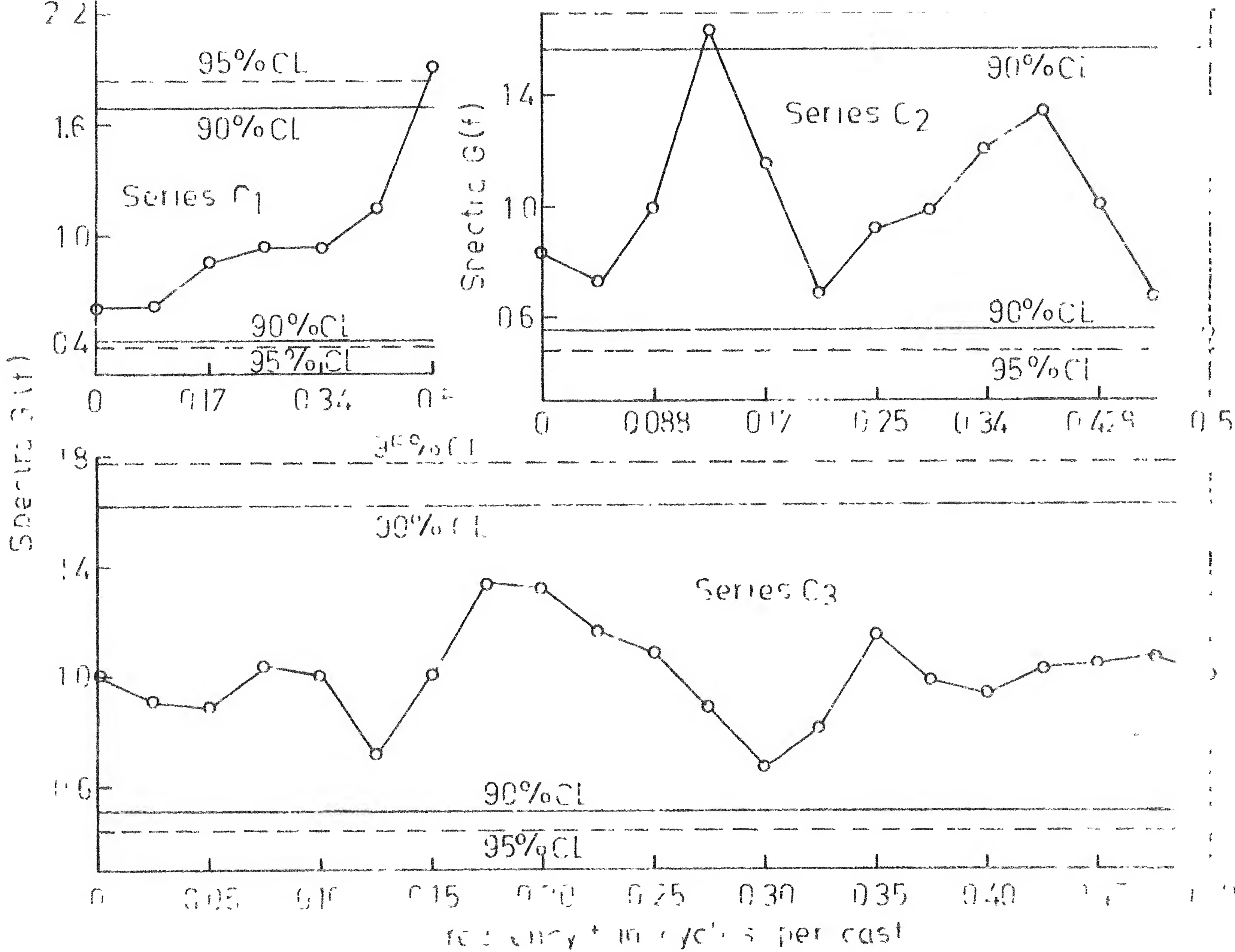
Fig 3.19c - Correlogram of univariate residuals D, E and F



3.20 - Power spectra of univariate residuals of sinter-to-coke ratio series A



3.21 - Power spectra of univariate residuals of blast run



Series C_1
95%CL
90%CL
90%CL
95%CL
2.2
1.6
1.0
0.4
0
0.17
0.34
Series C_2
90%CL
90%CL
95%CL
Spectra G(f)
1.4
1.0
0.6
0
0.088
0.25
0.34
0.5
Series C_3
90%CL
90%CL
95%CL
1.8
1.0
0
0.05
0.15
0.25
0.30
0.35
0.40
per cast

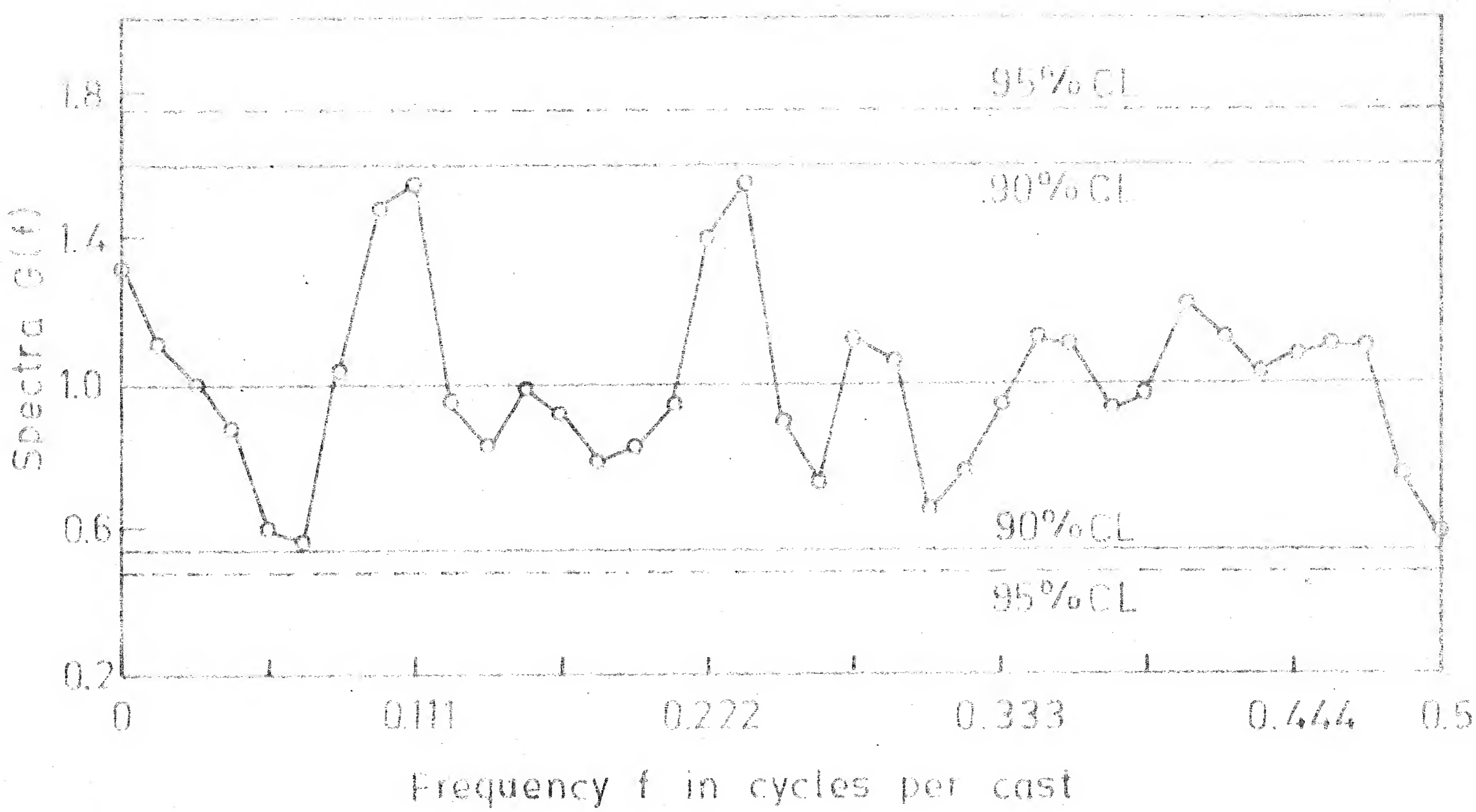


Fig. 3.23 - Power spectra of univariate residuals of hot metal temperature series D.

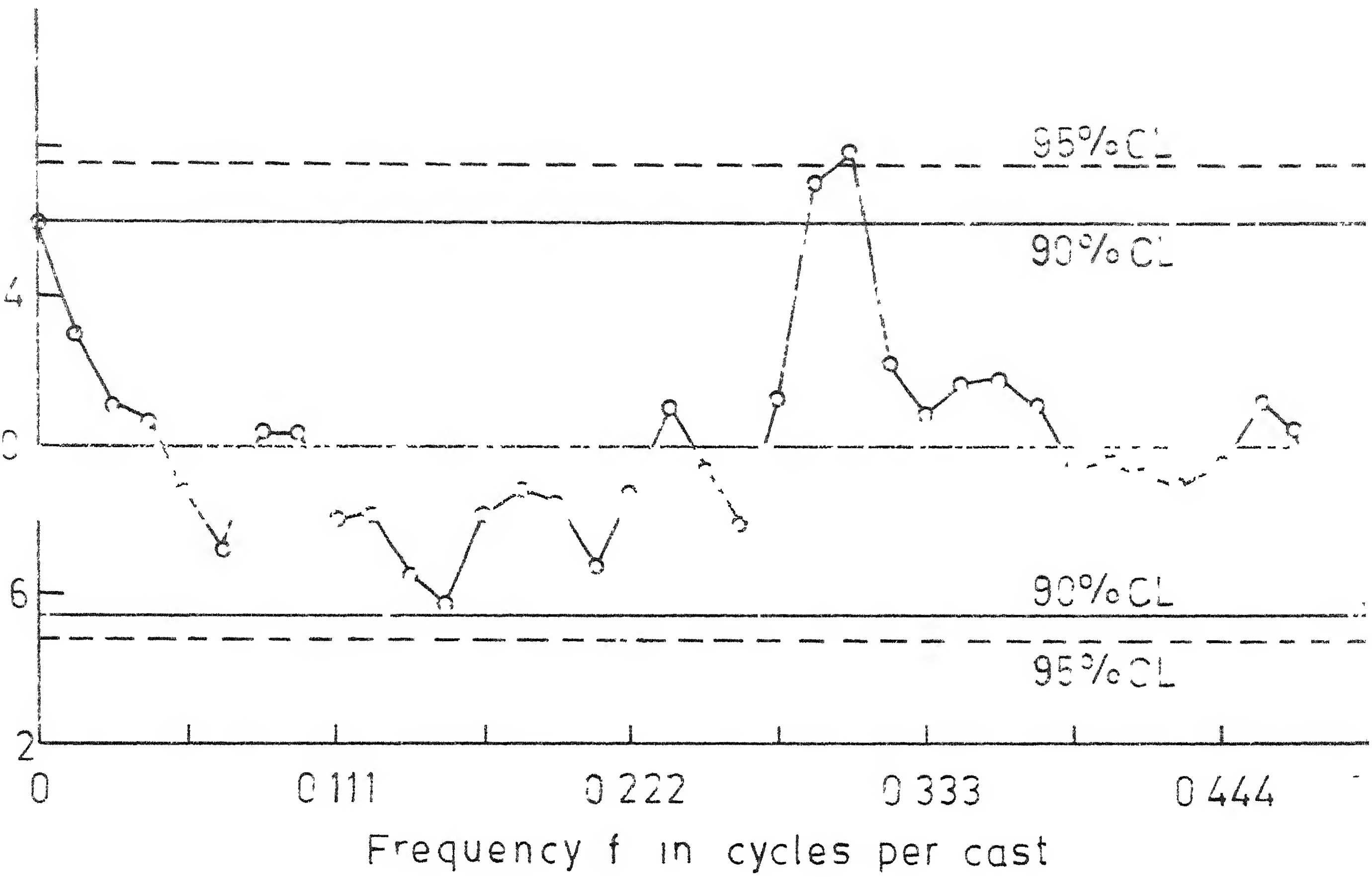


Fig 3 24 - Power spectra of univariate residuals of silicon content series E

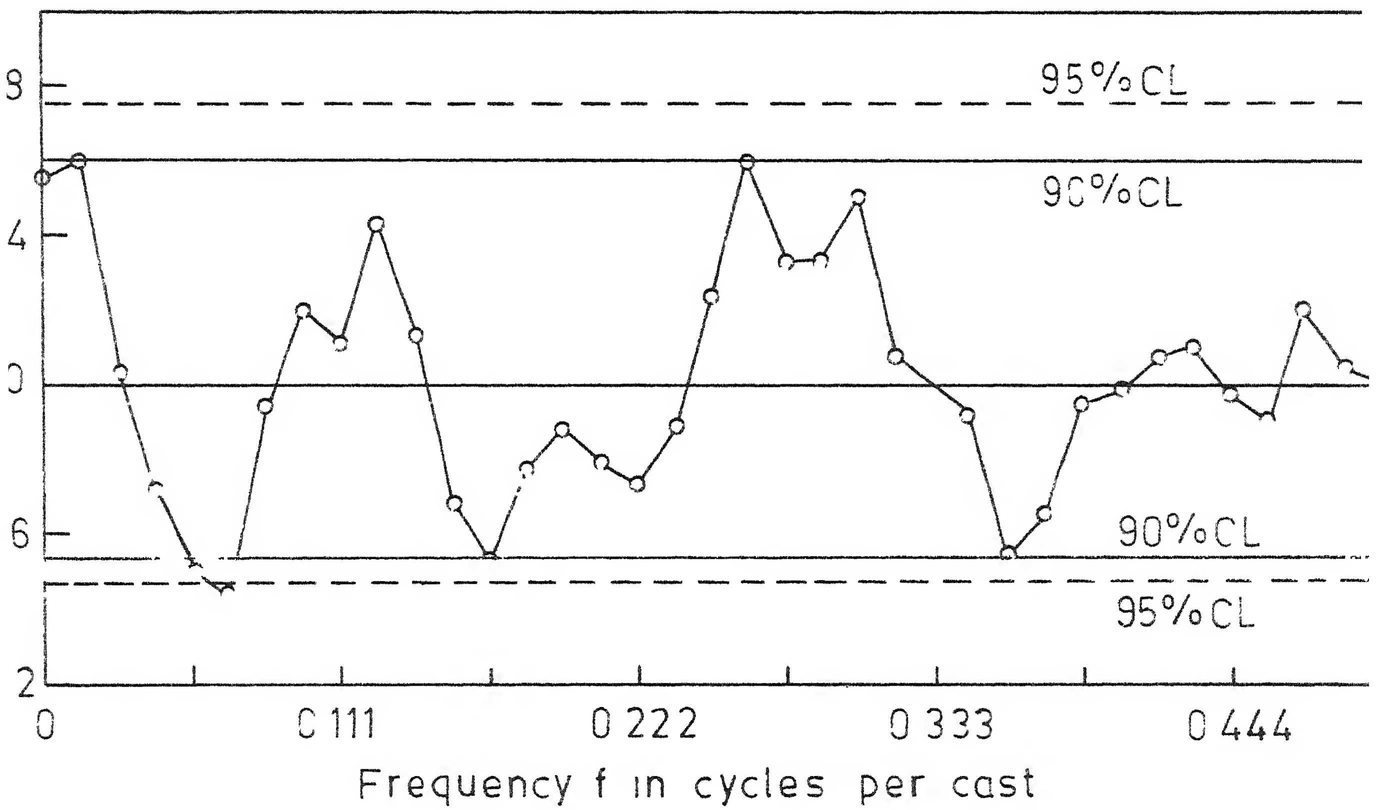


Fig 3 25 - Power spectra of univariate residuals of sulphur

Third order AR process:

$$G(f) = \frac{2\sigma_\varepsilon^2}{[(1-\varphi_1 \mathrm{Cos}2\pi f - \varphi_2 \mathrm{Cos}4\pi f - \varphi_3 \mathrm{Cos}6\pi f)^2 + (\varphi_1 \mathrm{Sin}2\pi f + \varphi_2 \mathrm{Sin}4\pi f + \varphi_3 \mathrm{Sin}6\pi f)^2]} \tag{3.64}$$

Mixed ARMA (1,0,1) Process:

$$G(f) = 2\sigma_\varepsilon^2 \; \frac{1+\theta_1^2 - 2\theta_1 \; \mathrm{Cos}2\pi f}{1+\varphi_1^2 - 2\varphi_1 \; \mathrm{Cos}2\pi f} \tag{3.65}$$

The confidence limit for power spectra is calculated using Eqn. 3.61, in which now G(f) stands for power spectra at a given frequency f of the series under consideration. The model is considered to be adequate at 95 per cent confidence level if power spectra generally lies within the confidence limit.

Using the models of best fit, power spectra and confidence limits were calculated for all series. The power spectra calculated using Eqn. 3.62 and confidence limits are also shown in Figures 3.13 to 3.18. In all the cases, the spectra of series lies within confidence limit which support the adequacy of the fitted models. The sample spectra is nearer the lower confidence limit than the upper one. This may be due to the maximum likelihood estimation procedure adopted in the study and other procedures for parameter estimation may lead to different results. In this study, the fitted models were considered to be adequate.

3.5.6 Stability of Univariate Models:

The stability of the fitted univariate models was tested by finding the roots of the characteristic equation 3.21. The characteristic equations and the corresponding roots have been given in Table 3.6. In all the cases, it was found that the roots of the characteristic equations lie within the unit circle. Hence all the fitted models are stable.

TABLE 3.6

STABILITY OF UNIVARIATE MODELS

Series	Order of the model	Characteristic Equation	Roots
A	(3,0,0)	$z^3-0.977z^2+0.197z-0.151=0$	0.940; $0.0185 \pm 0.4j$
B	(3,0,0)	$z^3-0.694z^2+0.004z-0.121=0$	0.855; $-0.0805 \pm 0.037j$
C_1	(1,0,0)	$z-0.897=0$	0.897
C_2	(1,0,1)	$z-0.282=0$	0.282
C_3	(3,0,0)	$z^3-0.684z^2+0.205z-0.344=0$	0.890; $-0.103 \pm 0.61j$
D	(2,0,0)	$z^2-0.237z-0.308=0$	0.687; -0.45
E	(1,0,0)	$z-0.473=0$	0.473
F	(2,0,1)	$z^2+0.222z-0.354=0$	0.716; -0.495

+++++++++

CHAPTER 4

MULTIVARIATE TIME SERIES ANALYSIS

The sequence of values of one variable constitutes a univariate time series. The sequences of values of a number of variables constitute a multivariate time series. The modelling and analysis of multivariate time series should take into account not only the serial dependence within each of the series but also the mutual or cross-correlation among the time series.

4.1 GENERAL MULTIVARIATE MODEL:

4.1.1 Autoregressive Moving Average (ARMA) Model:

Let the multivariate vector of the variables be given by

$$\underline{x}(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^T$$

where K is the total number of variables and T stands for the transpose. To represent the relationship among the time series, a general (p,q) order multivariate model of the following form may be considered:

$$\underline{x}(t+1) = \underline{\underline{A}}_1\ \underline{x}(t) + \ldots + \underline{\underline{A}}_p\ \underline{x}(t+1-p) + \underline{\underline{B}}_1\ \underline{E}(t+1) \ldots\ldots + \underline{\underline{B}}_q \underline{E}\ (t+2-q) \qquad (4.1)$$

where $\underline{\underline{A}}_i$, $i = 1,2,\ldots,p$ are the autoregressive (AR) coefficients matrices and $\underline{\underline{B}}_j$, $j=1,2,\ldots,q$ are the moving average (MA) coefficient matrices, all of dimension (KxK) and $\underline{E}(t)$ stands for the vector of residual series. Equation 4.1 can be considered as an extension of the univariate ARMA model to multivariate

domain. With adequate data, this equation can be solved for all unknowns. Since there are large number of parameters, the result may be data dependent or unstable.

4.1.2 Triangular Two Sided Moving Average (TTSMA) Model:

Since an autoregressive model can be expressed as an infinite order moving average model, the general ARMA model can also be expressed as an infinite order moving average model. Considering finite moving average terms, Phadke et al.[2] have proposed a triangular two sided moving average (TTSMA) model, which may be represented as follows:

$$\begin{aligned} \underline{x}(t) = {} & \underline{\underline{D}}_{-q_1}\underline{\eta}(t+q_1) + \underline{\underline{D}}_{-q_1+1}\underline{\eta}(t+q_1-1) + \ldots\ldots \\ & \ldots\ldots + \underline{\underline{D}}_{-1}\underline{\eta}(t+1) + \underline{\underline{D}}_{0}\underline{\eta}(t) + \underline{\underline{D}}_{1}\underline{\eta}(t-1) + \ldots\ldots \\ & \ldots\ldots \underline{\underline{D}}_{q_2-1}\,\underline{\eta}(t-q+1) + \underline{\underline{D}}_{q_2}\,\underline{\eta}(t-q_2) \end{aligned} \quad (4.2)$$

where $\underline{\eta}(t)$ is a vector of serially and mutually independent residual series and $\underline{\underline{D}}_i$, $i=-q_1, -q_1+1, \ldots, 0, 1, \ldots, q_2$ are lower triangular matrices. Equation 4.2 can also be written in terms of backshift operator B as follows:

$$\begin{aligned} \underline{x}(t) = {} & \underline{\underline{D}}_{-q_1} B^{-q_1}\,\underline{\eta}(t) + \underline{\underline{D}}_{-q_1+1}\,B^{-q_1+1}\,\underline{\eta}(t) + \ldots + \underline{\underline{D}}_{-1}B^{-1}\,\underline{\eta}(t) \\ & + \underline{\underline{D}}_{0}\,\underline{\eta}(t) + \underline{\underline{D}}_{1}\,B\,\underline{\eta}(t) + \ldots + \underline{\underline{D}}_{q_2-1}\,B^{q_2-1}\,\underline{\eta}(t) + \underline{\underline{D}}_{q_2}B^{q_2}\underline{\eta}(t) \end{aligned} \quad (4.3)$$

or $$\underline{x}(t) = \underline{\underline{\Psi}}(B)\,\underline{\eta}(t) \quad (4.4)$$

where $\underline{\underline{\Psi}}(B)$ is a lower triangular matrix of the form

$$\underline{\Psi}(B) = \begin{bmatrix} \Psi_{11}(B) & & & \\ \Psi_{21}(B) & \Psi_{22}(B) & & \\ & & & \\ \Psi_{K1}(B) & \Psi_{K2}(B) & \ldots\ldots\ldots & \Psi_{KK}(B) \end{bmatrix} \quad (4.5)$$

and

$\underline{\eta}(t) = [\eta_1(t),\ \eta_2(t)\ldots\ldots,\eta_K(t)]^T$ is a vector of K serially and mutually independent series.

4.2. DECOUPLED MULTIVARIATE MODEL:

In developing a decoupled multivariate model, initially a univariate stochastic model is fitted to each of the series, thus accounting for the internal correlation. The resulting residuals, which are serially independent and normally distributed are called 'prewhitened series'. A multivariate model is then fitted to the univariate residual series in terms of serially and mutually uncorrelated random components to explain external correlation. This approach for modelling of multivariate time series is advantageous because of the following reasons:

(i) Since serially independent random variates are used in the multivariate model, a better performance of the model can be expected than with the serially correlated data;

(ii) Since the parameters are estimated in stages, the number of parameters in the two stages are comparable respectively to that of univariate and multivariate models and the problem of simultaneously estimating all parameters is avoided, and

(iii) This procedure enables the most appropriate form and order of the univariate model to be chosen independently for each variable. The form and order of multivariate model are independent of the univariate models, Thus, it affords greater flexibility in the choice of appropriate but different models for the univariate and multivariate time series.

4.2.1. ARMA Model:

Recently Ramaseshan et al. [76] and Krishnasami [77] have proposed a decoupled multivariate model. According to this method, let $\underline{\varepsilon}(t)$ be a vector of serially independent random variables, obtained as residuals from univariate time series analysis. All residuals have mean zero and standard deviation $\hat{\sigma}_{\varepsilon k}$. The set of series $\varepsilon_k(t)$ is then standardized to a $\underline{E}(t)$ series with mean zero and unit standard deviation. Hence,

$$\underline{E}(t) = \left[\frac{\varepsilon_1(t)}{\hat{\sigma}_{\varepsilon_1}}, \frac{\varepsilon_2(t)}{\hat{\sigma}_{\varepsilon_2}}, \ldots\ldots, \frac{\varepsilon_K(t)}{\hat{\sigma}_{\varepsilon k}}\right]^T$$

$\underline{E}(t)$ constitutes the multivariate series of serially independent components and is related to the serially and mutually uncorrelated series by a multivariate ARMA model of order (p,q), as follows

$$\underline{E}(t) = \sum_{u=1}^{p} \underline{\underline{C}}_u \, \underline{E}\,(t-u) + \sum_{v=1}^{q} \underline{\underline{D}}_v \, \underline{\eta}(t+1-v) \qquad (4.6)$$

If u=v=1, Eqn. 4.6 becomes

$$\underline{E}(t) = \underline{\underline{C}}_1 \, \underline{E}(t-1) + \underline{\underline{D}}_1 \, \underline{\eta}(t) \qquad (4.7)$$

Let $\underline{\underline{M}}_0$ and $\underline{\underline{M}}_{-1}$ be lag zero and lag one correlation matrices.

In order to preserve these correlations the coefficients of the model given by Eqn. 4.7 can be estimated from

$$\underline{\underline{C}}_1 = \underline{\underline{M}}_{-1}\, \underline{\underline{M}}_o^{-1} \qquad (4.8)$$

and

$$\underline{\underline{D}}_1\, \underline{\underline{D}}_1^T = \underline{\underline{M}}_o - \underline{\underline{C}}_1\, \underline{\underline{M}}_{-1}^T \qquad (4.9)$$

This is essentially similar to the method of moments. The matrix $\underline{\underline{D}}_1$ is a lower triangular matrix. In this method all elements of matrix $\underline{\underline{C}}_1$ and all lower triangular elements of the matrix $\underline{\underline{D}}_1$ are to be estimated whether they are statistically significant or not. However, some of the elements may be statistically insignificant and it is not necessary to estimate all elements, in such case. Secondly Eqns. 4.8 and 4.9 are used when the multivariate modelis first order autoregressive and first order moving average. If the order of the model is higher than (1,1), the estimation of parameters becomes more complicated because a large number of them have to be estimated and moreover, these estimates may not be reliable.

4.2.2 <u>TTSMA Model</u>:

As in case of general multivariate model, the multivariate ARMA model given by Eqn. 4.6 may be expressed as an infinite order moving average model. Phadke et al. [2] have developed a TTSMA model of finite moving average order which is given by

$$\underline{\varepsilon}(t) = \underline{\underline{\Psi}}'(B)\, \underline{\eta}\, (t) \qquad (4.10)$$

where $\underline{\varepsilon}(t) = [\varepsilon_1(t), \varepsilon_2(t), \ldots, \varepsilon_k(t)]^T$ is the vector of

univariate residuals and $\underline{\underline{\Psi}}^{*}(B)$ is the lower triangular matrix of the form given by Eqn. 4.5. The advantage of this method over previous method is that all elements of the lower triangular matrix $\underline{\underline{\Psi}}^{*}(B)$ need not have to be estimated. The statistical significance of the elements is obtained by plotting cross-correlation function between $\underline{\varepsilon}(t)$ and $\underline{\eta}(t)$. Hence, only those elements which are significant have to be estimated, say, by the method of least squares.

The block diagram of Eqn. 4.10 is given in Figure 4.1. Thus,

$$\varepsilon_1(t) = \Psi^{*}_{11}(B)\ \eta_1(t) \tag{4.11}$$

The series $\varepsilon_2(t)$ consists of two parts, $\varepsilon_{2.1}(t)$ and $\varepsilon_{2.2}(t)$,

$$\varepsilon_2(t) = \varepsilon_{2.1}(t) + \varepsilon_{2.2}(t) \tag{4.12}$$

Out of these, $\varepsilon_{2.1}(t)$ is the projection of $\varepsilon_2(t)$ series on the space of $\eta_1(t)$ series and has the model

$$\varepsilon_{2.1}(t) = \Psi^{*}_{21}(B)\ \eta_1(t) \tag{4.13}$$

The other component $\varepsilon_{2.2}(t)$ is orthogonal to $\eta_1(t)$ series and has the model

$$\varepsilon_{2.2}(t) = \Psi^{*}_{22}(B)\ \eta_2(t) \tag{4.14}$$

Thus,

$$\varepsilon_2(t) = \Psi^{*}_{21}(B)\ \ \eta_1(t) + \Psi^{*}_{22}(B)\ \eta_2(t) \tag{4.15}$$

In general, the series $\varepsilon_j(t)$ consists of j orthogonal components, $\varepsilon_{j\ k}(t) = \Psi_{j\ k}(B)\ \eta_k(t)$, $k = 1,2,\ldots,j$. The first j-1 components stand for the projection of $\varepsilon_j(t)$ on the space of

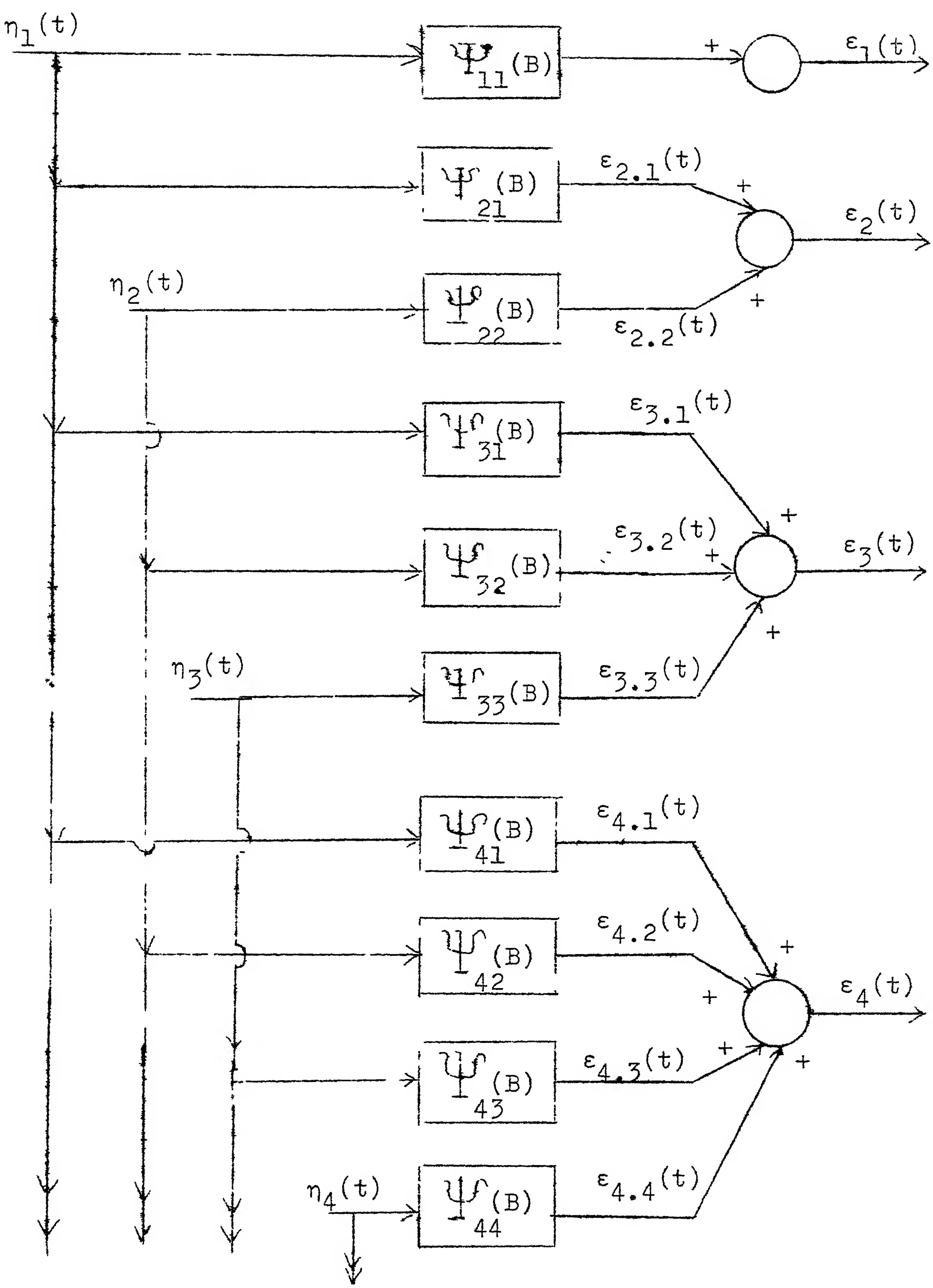


FIG.4.1 BLOCK DIAGRAM OF TTSMA MODEL
(Adapted from Phadke et al.[2])

$\eta_k(t)$, $k=1,2,\ldots,j-1$. The jth component $\varepsilon_{j\cdot j}(t)$ defines a new whitenoise series $\Psi_{jj}(B)\,\eta_j(t)$ orthogonal to $\Psi_{kk}(B)\eta_k(t)$, $k=1,2,\ldots,j-1$. Hence, this model represents Gram-Schmidt orthogonalization of stationary multivariate time series $\underline{\varepsilon}(t)$ into K orthogonal vectors $\Psi_{kk}(B)\eta_k(t)$, $k=1,2,\ldots K$.

The method adopted by Phadke et al to identify and estimate the parameters of the matrix $\underline{\underline{\Psi}}(B)$ is as follows:

(i) Since $\varepsilon_1(t)$ is the residual series obtained by univariate modelling of the series, $x_1(t)$, it is a prewhitened series. Hence, the Eqn. 4.11 takes the form

$$\varepsilon_1(t) = \eta_1(t) \tag{4.16}$$

(ii) $\Psi_{21}(B)$ is identified by plotting the cross-correlation function between $\varepsilon_2(t)$ and $\eta_1(t)$. Approximate estimates of $\Psi_{21}(B)$ are obtained from

$$\Psi_{21}\ (B) = \frac{r_{\varepsilon_2\eta_1}(B)\ \ (\mathrm{Var}\ \varepsilon_2(t))^{\frac{1}{2}}}{(\mathrm{Var}\ \eta_1(t))^{\frac{1}{2}}} \tag{4.17}$$

in which $r_{\varepsilon_2\eta_1}(B)$ stands for the cross-correlation function between $\varepsilon_2(t)$ and $\eta_1(t)$ series at lag B. $\varepsilon_{2.1}(t)$ is then calculated from Eqn. 4.13,

$$\varepsilon_{2.1}(t) = \Psi_{21}(B)\ \eta_1(t) \tag{4.13}$$

(iii) On subtracting $\varepsilon_{2.1}(t)$ from $\varepsilon_2(t)$ gives the series $\varepsilon_{2.2}(t)$ according to Eqn. 4.12.

(iv) $\varepsilon_{2.2}(t)$ is expressed as Eqb. 4.14 and the parameters of $\Psi_{22}(B)$ are estimated by univariate modelling of time series.

This procedure is repeated for all series. That is,

(a) Calculate cross-correlation function between $\varepsilon_i(t)$ and $\eta_j(t)$, $j = 1,2,\ldots,i-1$; and hence identify $\Psi_{ij}(B)$.

(b) Multiply $\varepsilon_i(t)$ by corresponding $\Psi_{ij}(B)$, $j=1,\ldots,i-1$ to give the $(i-1)$ components of $\varepsilon_i(t)$ which are nothing but projections of $\varepsilon_i(t)$ on $\eta_j(t)$, $j=1,2,\ldots,i-1$.

(c) Subtract these components from $\varepsilon_i(t)$ to give the series $\varepsilon_{ii}(t)$ which is orthogonal to $\eta_j(t)$, $j=1,2,\ldots i-1$.

(d) Identify and estimates $\Psi_{ii}(B)$ by univariate modelling of $\varepsilon_{ii}(t)$ series.

(e) Finally estimate the parameters of $\Psi_{ij}(B)$, $j=1,2,\ldots i-1$ by the method of least squares.

In the above method, the Gram-Schmidt orthogonalization process has not been used directly. Secondly, in identifying $\Psi_{ij}(B)$, the cross-correlation function was calculated between $\varepsilon_i(t)$ and $\eta_j(t)$. Now, $\varepsilon_i(t)$ consists of i components. In particular, the presence of noise $\eta_i(t)$ or $\varepsilon_{i\cdot i}(t) = \Psi_{ii}(B)\eta_i(t)$ corrupts the cross-correlation function between $\varepsilon_i(t)$ and $\eta_j(t)$. Hence, in the present study, the above method of developing the matrix $\underline{\underline{\Psi}}(B)$ has been modified.

4.2.3 Generalized Method for Parameter Estimation:

The method of building the multivariate model can also be generalized which leads to a simplified method for parameter estimation. From the set of vectors $\underline{\varepsilon}(t)$ one can obtain a set of orthogonal vectors $\underline{\beta}(t)$ by means of Gram-Schmidt orthogonalization. The algorithm [78] is as follows, and details of

which are given in Appendix **B**.

$$\beta_1(t) = \varepsilon_1(t) \tag{4.18}$$

$$\alpha_1(t) = \frac{\beta_1(t)}{\sqrt{(\beta_1(t)\cdot\beta_1(t))}} = \frac{\beta_1(t)}{\|\beta_1(t)\|} \tag{4.19}$$

$$\beta_j(t) = \varepsilon_j(t) - \sum_{i=1}^{j-1} (\alpha_i(t)\cdot\varepsilon_j(t))\ \alpha_i(t) \tag{4.20}$$

$$1 < j \leq K$$

$$\alpha_i(t) = \frac{\beta_i(t)}{\sqrt{(\beta_i(t)\cdot\beta_i(t))}} = \frac{\beta_i(t)}{\|\beta_i\ (t)\|} \tag{4.21}$$

in which $\alpha_i(t)$ represents ith orthonormal vector and $\|\beta_i(t)\|$ stands for the norm of the vector $\beta_i(t)$, $(\alpha_i(t)\cdot\varepsilon_j(t))$ stands for the inner product of $\alpha_i(t)$ and $\varepsilon_j(t)$. Now writing Eqn. 4.20 for $\beta_2(t)$,

$$\beta_2(t) = \varepsilon_2(t) - (\alpha_1(t)\cdot\varepsilon_2(t))\ \alpha_1(t) \tag{4.22}$$

or

$$\beta_2(t) = \varepsilon_2(t) - \delta_{12}\cdot\alpha_1(t) \tag{4.23}$$

where $\delta_{12} = (\alpha_1(t)\cdot\varepsilon_2(t))$

Substituting for $\alpha_1(t)$ from Eqn. 4.19 in 4.23

$$\beta_2(t) = \varepsilon_2(t) - (\delta_{12}/\|\beta_1(t)\|)\ \beta_1(t) \tag{4.24}$$

which may be written as

$$\varepsilon_2(t) = H_{21}\ \beta_1(t) + \beta_2(t) \tag{4.25}$$

where

$$H_{21} = \frac{\delta_{12}}{\|\beta_1(t)\|} \tag{4.26}$$

Similarly,

$$\beta_3(t) = \varepsilon_3(t) - (\alpha_1(t)\cdot\varepsilon_3(t))\ \alpha_1(t) - (\alpha_2(t)\cdot\varepsilon_3(t))\ \alpha_2(t) \tag{4.27}$$

which when combined with Eqn. 4.21 becomes

$$\beta_3(t) = \varepsilon_3(t) - \frac{\delta_{13}}{\|\beta_1(t)\|}\ \beta_1(t) - \frac{\delta_{23}}{\|\beta_2(t)\|}\ \beta_2(t) \tag{4.28}$$

where $\delta_{13} = (\alpha_1(t)\cdot\varepsilon_3(t))$

$\delta_{23} = (\alpha_2(t)\cdot\varepsilon_3(t))$

or
$$\varepsilon_3(t) = H_{31}\ \beta_1(t) + H_{32}\ \beta_2(t) + \beta_3(t) \tag{4.29}$$

where
$$H_{31} = \delta_{13}/\|\beta_1(t)\|$$
$$H_{32} = \delta_{23}/\|\beta_2(t)\| \tag{4.30}$$

In general,

$$\varepsilon_j(t) = H_{j1}\ \beta_1(t) + \ldots + H_{j\cdot j-1}\ \ \beta_{j-1}(t) + \beta_j(t) \tag{4.31}$$

where
$$H_{jk} = (\alpha_k(t)\cdot\varepsilon_j(t))/\ \|\beta_k(t)\|$$
$$= \delta_{kj}/\|\beta_k(t)\| \tag{4.32}$$

The Eqn. 4.31 can be written for all variables from $j=1,2,\ldots K$ as follows:

$$\begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \varepsilon_j(t) \\ \\ \varepsilon_K(t) \end{bmatrix} = \begin{bmatrix} 1 & & & \\ H_{21} & 1 & & \\ H_{j1} & H_{j\cdot 2}\cdots\cdots H_{j\cdot j-1} & 1 & \\ \\ H_{K1} & H_{K2}\cdots\cdots\cdots\cdots H_{K\cdot K-1} & & 1 \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \\ \beta_j(t) \\ \\ \beta_K(t) \end{bmatrix} \tag{4.33}$$

or $\underline{\varepsilon}(t) = \underline{\underline{H}}\ \underline{\beta}(t)$ (4.34)

This is known as a principal component model.

Since $\underline{\beta}(t)$ is a set of orthogonal vectors, the dot product between any two series $\beta_i(t)$ and $\beta_j(t)$, $i \neq j$ will be zero. Hence, cross-correlation function at lag zero will be zero, however, cross-correlation function between $\beta_i(t)$ and $\beta_j(t)$ may be significant at lags other than zero. Let $r_{\beta_2\beta_1}(k)$ be the cross-correlation coefficient between series $\beta_2(t)$ and $\beta_1(t)$ series at lag k. From the cross-correlogram, the coefficients which are significant at 95 per cent confidence level are determined. Let $r_{\beta_2\beta_1}(B)$ represent the significant cross-correlation coefficients at lag B, then a quantity $\lambda_{21}(B)$ is calculated from

$$\lambda_{21}(B) = \frac{r_{\beta_2\beta_1}(B)\ (\mathrm{Var}\ \beta_2(t))^{\frac{1}{2}}}{(\mathrm{Var}\ \beta_1(t))^{\frac{1}{2}}} \tag{4.35}$$

$\varepsilon_{2.1}(t)$ is then calculated from

$$\varepsilon_{2.1}(t) = H_{21} \cdot \beta_1(t) + \lambda_{21}(B)\ \ \beta_1(t) \tag{4.36}$$

and is subtracted from $\varepsilon_2(t)$ to give $\varepsilon_{2.2}(t)$ series. $\varepsilon_{2.2}(t)$ is then expressed as Eqn. 4.14 and the parameters of $\Psi_{22}(B)$ are estimated by univariate modelling of time series. Thus $\varepsilon_2(t)$ can be expressed as

$$\begin{aligned}\varepsilon_2(t) &= H_{21}\beta_1(t) + \lambda_{21}(B)\ \beta_1(t) + \Psi_{22}(B)\ \eta_2(t) \\ &= (H_{21} + \lambda_{21}(B))\beta_1(t) + \Psi_{22}(B)\ \eta_2(t)\end{aligned} \tag{4.37}$$

or $\varepsilon_2(t) = \Psi_{21}(B)\ \eta_1(t) + \Psi_{22}(B)\ \eta_2(t)$

The parameters of the model given by Eqn. 4.37 are estimated by multiple linear regression analysis. The method of centralization and correlation matrix [79] has been used for regression analysis. This procedure is repeated successively for all series.

4.3. TRANSFER FUNCTION MODEL:

The decoupled multivariate model developed by Phadke et al. is useful in determing the transfer function model for a physical system under control. Consider a stable, linear, closed loop system which has m-manipulatable variables, $X_j(t)$, $j=1,\ldots,m$, and n-output variables $Y_i(t)$, $i=1,\ldots.n$. Then letting $\underline{X}(t)$ and $\underline{Y}(t)$ to represent corresponding m-dimensional input vector and n-dimensional output vector, the model for the plant can be written as

$$\underline{Y}(t) = \underline{\underline{V}}(B)\ \underline{X}(t) + \underline{N}(t) \qquad (4.38)$$

where $\underline{\underline{V}}(B)$ is a nxm matrix which represents plant dynamics. Every element $V_{ij}(B)$ of $\underline{\underline{V}}(B)$ represents transfer function between the output $Y_i(t)$ and the input $X_j(t)$, and has the form

$$V_{ij}\ (B) = B^{b_{ij}} \frac{w_{ij}(o) + w_{ij}(1)\ B + \ldots . + w_{ij}(s_{ij})\ B^{s_{ij}}}{1+\delta'_{ij}(1)\ B + \ldots .. + \delta'_{ij}(r_{ij})\ B^{r_{ij}}} \qquad (4.38a)$$

where b^{ij} represents the time delay or dead time between the input $X_j(t)$ and output $Y_i(t)$.

$\underline{N}(t)$ represents the plant disturbance, which is n-dimensional, nonsingular stationary process and can be described by

$$\underline{N}(t) = \underline{\underline{\Psi}}'(B) \quad \underline{\zeta}'(t) \qquad (4.39)$$

where $\underline{\zeta}'(t)$ is n-dimensional white noise process such that its variance-covariance matrix $E[(\underline{\zeta}'(t))(\underline{\zeta}'(t))^T] = \sum_{\zeta'}$ is a diagonal matrix.

Let the characteristics of the feed back dynamics matrix $\underline{\underline{C}}(B)$, a mxn matrix, be similar to those of $\underline{\underline{V}}(B)$; then the model for feedback can be written as

$$\underline{X}(t) = \underline{\underline{C}}(B) \ \underline{Y}(t) + \underline{M}(t) \qquad (4.40)$$

where $\underline{M}(t)$ represents m-dimensional, nonsingular, stationary feed back disturbance and can be described by

$$\underline{M}(t) = \underline{\underline{\Psi}}''(B) \quad \underline{\zeta}''(t) \qquad (4.41)$$

in which $\underline{\zeta}''(t)$ represents m-dimensional white noise process such that its variance-covariance matrix is a diagonal matrix.

If I_m and I_n represent mth order and nth order unit matrices, respectively, then Eqns. 4.38 and 4.40 can be written as

$$[\underline{\underline{I}}_m - \underline{\underline{C}}(B) \ \underline{\underline{V}}(B)] \ \underline{X}(t) = \underline{M}(t) + \underline{\underline{C}}(B) \ \underline{N}(t) \qquad (4.42)$$

$$[\underline{\underline{I}}_n - \underline{\underline{V}}(B) \ \underline{\underline{C}}(B)] \ \underline{Y}(t) = \underline{\underline{V}}(B) \ \underline{M}(t) + \underline{N}(t) \qquad (4.43)$$

If $\underline{\underline{D}}_1(B) = [\underline{\underline{I}}_m - \underline{\underline{C}}(B) \ \underline{\underline{V}}(B)]^{-1}$

and $\underline{\underline{D}}_2(B) = [\underline{\underline{I}}_n - \underline{\underline{V}}(B) \ \underline{\underline{C}}(B)]^{-1}$ (4.44)

Then Eqns. 4.42 and 4.43 become

$$\underline{X}(t) = \underline{\underline{D}}_1(B)\, \underline{\underline{\Psi}}''(B)\, \underline{\zeta}''(t) + \underline{\underline{D}}_1(B)\, \underline{\underline{C}}(B)\, \underline{\underline{\Psi}}'(B)\, \underline{\zeta}'(t) \tag{4.45}$$

$$\underline{Y}(t) = \underline{\underline{D}}_2(B)\, \underline{\underline{V}}(B)\, \underline{\underline{\Psi}}''(B)\, \underline{\zeta}''(t) + \underline{\underline{D}}_2(B)\, \underline{\underline{\Psi}}'(B)\, \underline{\zeta}'(t) \tag{4.46}$$

Let

$$\underline{\underline{L}}_{11}(B) = \underline{\underline{D}}_1(B)\, \underline{\underline{\Psi}}''(B) \quad , \quad (m \times m) \tag{4.47}$$

$$\underline{\underline{L}}_{12}(B) = \underline{\underline{D}}_1(B)\, \underline{\underline{C}}(B)\, \underline{\underline{\Psi}}'(B) \quad , \quad (m \times n) \tag{4.48}$$

$$\underline{\underline{L}}_{21}(B) = \underline{\underline{D}}_2(B)\, \underline{\underline{V}}(B)\, \underline{\underline{\Psi}}''(B) \quad , \quad (n \times m) \tag{4.49}$$

and

$$\underline{\underline{L}}_{22}(B) = \underline{\underline{D}}_2(B)\, \underline{\underline{\Psi}}'(B) \quad , \quad (n \times n) \tag{4.50}$$

Then Eqns. 4.45 and 4.46 can be written as

$$\underline{X}(t) = \underline{\underline{L}}_{11}(B)\, \underline{\zeta}''(t) + \underline{\underline{L}}_{12}(B)\, \underline{\zeta}'(t) \tag{4.51}$$

$$\underline{Y}(t) = \underline{\underline{L}}_{21}(B)\, \underline{\zeta}''(t) + \underline{\underline{L}}_{22}(B)\, \underline{\zeta}'(t) \tag{4.52}$$

which may be combined to give

$$\underline{Z}(t) = \underline{\underline{L}}(B)\, \underline{u}(t) \tag{4.53}$$

where $\underline{Z}(t) = (\underline{X}(t), \underline{Y}(t))^T$

and $\underline{u}(t) = (\underline{\zeta}''(t), \underline{\zeta}'(t))^T$

The variance-covariance matrix, $\Sigma_{\zeta''}$ of white noise series $\underline{\zeta}''(t)$ is the mxm principal minor of Σ_u, the variance-variance matrix of $\underline{u}(t)$, standing in the upper left corner. The nxn principal minor of Σ_u, standing in the lower right corner is equal to $\Sigma_{\zeta'}$, the variance-covariance matrix of $\underline{\zeta}'(t)$

The matrix $\underline{\underline{L}}(B)$ contains terms involving only positive powers of B. However, the elements $\psi_{jk}(B)$, $j > k$ of the matrix $\underline{\underline{\Psi}}(B)$ of TTSMA model may have poles outside as well as inside the unit circle. The matrix $\underline{\underline{L}}(B)$ may be obtained from TTSMA model by removing poles and zeros inside the unit circle by using shift transformation given in [80]. If raw (original) data are used in the multivariate analysis, then $\underline{Z}(t)$ of Eqn. 4.53 is equal to $\underline{x}(t)$ of general TTSMA model given by Eqn. 4.3. If the prewhitened series are used for multivariate analysis, then $\underline{Z}(t)$ is equal to $\underline{\varepsilon}(t)$ of Eqn. 4.10.

The matrices $\underline{\underline{V}}(B)$, $\underline{\underline{C}}(B)$, $\underline{\underline{\Psi}}'(B)$ and $\underline{\underline{\Psi}}''(B)$ are obtained from the matrix $\underline{\underline{L}}(B)$ as follows:

(i) Partition the matrix $\underline{\underline{L}}(B)$ as follows:

$$\underline{\underline{L}}(B) = \left[\begin{array}{c|c} L_{11}(B) & L_{12}(B) \\ \hline L_{21}(B) & L_{22}(B) \end{array}\right]$$

where the dimensions of submatrices are given in Eqns. 4.47 to 4.50.

(ii) Combining Eqns. 4.49 and 4.50

$$\underline{\underline{V}}(B) = \underline{\underline{\Psi}}'(B)\ \underline{\underline{L}}_{22}^{-1}(B)\ \ \underline{\underline{L}}_{21}(B)\ \ \underline{\underline{\Psi}}''^{-1}(B) \qquad (4.54)$$

similarly Eqns. 4.47 and 4.48 give

$$\underline{\underline{C}}(B) = \underline{\underline{\Psi}}''(B)\ \ \underline{\underline{L}}_{11}^{-1}(B)\ \ \underline{\underline{L}}_{12}(B)\ \underline{\underline{\Psi}}'^{-1}(B) \qquad (4.55)$$

(iii) From Eqns. 4.50 and 4.44 one gets

$$\underline{\underline{\Psi}}'(B) = \underline{\underline{L}}_{22}(B) - \underline{\underline{V}}(B)\ \underline{\underline{C}}(B) \qquad (4.56)$$

which on combining with Eqns. 4.54 and 4.55 gives

$$\underline{\underline{\Psi}}^{\prime}(B) = \underline{\underline{L}}_{22}(B) - \underline{\underline{L}}_{21}(B) \quad \underline{\underline{L}}_{11}^{-1}(B) \quad \underline{\underline{L}}_{12}(B) \tag{4.57}$$

Similarly Eqns. 4.47, 4.44, 4.54 and 4.55 give

$$\underline{\underline{\Psi}}^{\prime\prime}(B) = \underline{\underline{L}}_{11}(B) - \underline{\underline{L}}_{12}(B) \; \underline{\underline{L}}_{22}^{-1}(B) \; \underline{\underline{L}}_{21}(B) \tag{4.58}$$

Thus, from the multivariate model, various transfer functions (i.e., for plant, feed back, plant noise and feed backn noise), canbe obtained.

4.4 ANALYTICAL PROCEDURES:

In order to identify and estimate the parameters of the multivariate model (Eqn. (4.10) it is necessary to calculate cross-correlation functions between the series $\underline{\varepsilon}(t)$ and $\underline{\eta}(t)$. Just as autocorrelation function and the spectral density function are helpful to identify univariate time series models, the cross-correlation function and cross spectral density function are helpful in identification of the multivariate models.

4.4.1 Cross-Correlation Function:

It determines the linear dependence between two series. The cross-covariance function between two series x(t) and y(t) at any lag k is given by

$$\gamma_{xy}(k) = \mathrm{cov}(x(t)\cdot y(t+k)) = E[(x(t)-\mu_x)(y(t+k)-\mu_y)]$$

The theoretical cross correlation coefficient at lag k is defined by

$$\hat{\rho}_{xy}(k) = \frac{\mathrm{cov}(x(t)\cdot y(t+k))}{(\mathrm{var}\ x(t)\cdot \mathrm{var}\ y(t+k))^{\frac{1}{2}}} \simeq \frac{\gamma_{xy}(k)}{\sigma_x\ \sigma_y} \qquad (4.59)$$

$$k=0,\pm 1,\pm 2,\ldots\ldots$$

Unlike the autocorrelation function, in general, cross-correlation function is not symmetrical about k=o that is $\hat{\rho}_{xy}(k)$ is not necessarily equal to $\hat{\rho}_{xy}(-k)$.

The cross-correlation function is usually estimated from

$$r_{xy}(k) = \frac{\frac{1}{N-k}\sum_{i=1}^{N-k} x(i)\ y(i+k) - \frac{1}{(N-k)^2}\sum_{i=1}^{N-k} x(i)\sum_{i=1}^{N-k} y(i+k)}{(\mathrm{var}\ x(i))^{\frac{1}{2}}\ (\mathrm{var}\ y(i+k))^{\frac{1}{2}}} \qquad (4.60)$$

where $\mathrm{var}x(i) = \frac{1}{N-k}\sum_{i=1}^{N-k} x(i)^2 - \frac{1}{(N-k)^2}\left[\sum_{i=1}^{N-k} x(i)\right]^2$

$\mathrm{var}\ y(i+k) - \frac{1}{N-k}\sum_{i=1}^{N-k} y(i+k)^2 - \frac{1}{(N-k)^2}\left[\sum_{i=1}^{N-k} y(i+k)\right]^2$

<u>Standard Error of Cross-Correlation Coefficient:</u> A crude check as to whether the cross-correlation coefficient $\hat{\rho}_{xy}(k)$ is not different from zero is made by comparing the corresponding estimate of the cross-correlation coefficient with their approximate standard errors obtained by a formula given by Bartlett.

$$SE[r_{xy}(k)] \simeq (N-k)^{-\frac{1}{2}} \qquad (4.61)$$

which is used to test the significance of cross correlation between two white noise processes. All cross-correlation coefficients beyond $\pm$ 1.96 times the standard error are considered to be

signicant at 95 per cent confidence level. The two series are said to be mutually independent if the cross-correlation coefficients lie generally within ± 1.96 times the standard error.

4.4.2 Cross Spectrum:

The correlation between any two series in the frequency domain is represented by the cross spectrum. The sample cross spectrum contains two different types of information about the dependence between the two processes. It is a complex quantity and may be written as product of a real function called the sample cross amplitude spectrum and a complex function called sample phase spectrum. The cross amplitude spectrum shows whether frequency components of one series are associated with large or small amplitudes of the other series of the same frequency. The cross phase spectrum shows whether frequency components of one series lag or lead the components of the same frequency of the other series. The cross spectrum is obtained by taking Fourier transformation of the cross-covariance function. In the present study, only cross-correlation function is used for identification purposes.

4.5 ANALYSIS AND DISCUSSION OF RESULTS:

As series C is nonstationary (Subsection 3.5.2) it was divided into three parts and, after standardization, different models were fitted to the parts separately. The residuals obtained from each part had zero mean and different variances. Hence, residuals of series C as a whole is nonstationary. In developing a multivariate model two ways may be adopted.
(i) All the series may be divided into three parts as series C and fit a multivariate model to each of the parts, and (ii) Standardise residuals of each part of the series C so that series C as a whole has zero mean and unit standard deviation. In the present study, the latter procedure is used. The standardized series C, along with residuals A,B,D,E and F constitute the multivariate time series. In multivariate time series analysis, the series were used in the order A, B,C,D,E and F.

4.5.1 Generalized Method:

This method is described in Subsection 4.2.3. Table 4.1 shows norms of the orthogonal vectors $\beta_k(t)$, $k=1,\ldots,6$; inner products of $\alpha_k(t)$ and $\varepsilon_j(t)$; and the parameters H_{jk}, of the principal component model (Eqn. 4.32). The autocorrelation functions of all the orthogonal vectors have been plotted in Figures 4.2(a) and 4.2(b). It is found that all orthogonal vectors are pure random series. In order to identify $\Lambda_{ij}(B)$, $i=1,\ldots,6$, $j=1,\ldots,6$, $i > j$; the cross-correlation functions

TABLE 4.1

PARAMETER ESTIMATES OF PRINCIPAL COMPONENT MODEL

k	j	Norms of $\beta_k(t)$ $\beta_k(t)$	Inner product $(\alpha_k(t) \cdot \varepsilon_j(t))$	$H_{jk} = \frac{\alpha_k(t) \cdot \varepsilon_j(t)}{\|\beta_k(t)\|}$
1	2	7.06	0.1786	0.0252
1	3	7.06	3.9006	0.5512
2	3	12.00	-0.9576	-0.0798
1	4	7.06	0.9843	0.1390
2	4	12.00	0.7777	0.0648
3	4	18.25	-0.5932	-0.0324
1	5	7.06	0.2548	0.0360
2	5	12.00	2.0611	0.1718
3	5	18.25	-5.5208	-0.3021
4	5	16.90	0.4034	0.0238
1	6	7.06	0.6053	0.0855
2	6	12.00	-1.1507	-0.0960
3	6	18.25	0.9659	0.0528
4	6	16.90	-2.0189	-0.1190
5	6	14.96	-7.5162	-0.5000

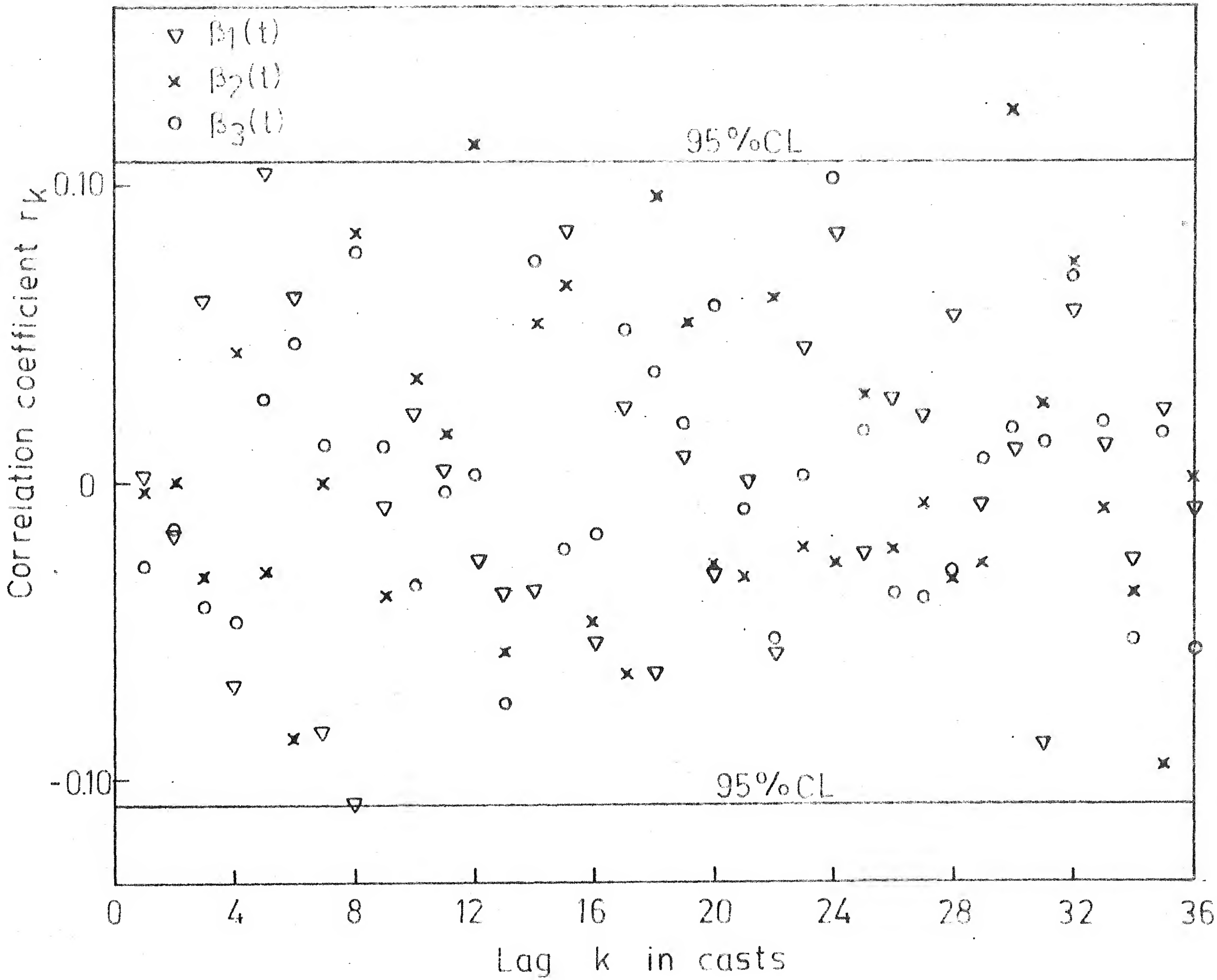


Fig. 4.2a - Correlogram of the orthogonal vectors $\beta_1(t)$ to $\beta_3(t)$.

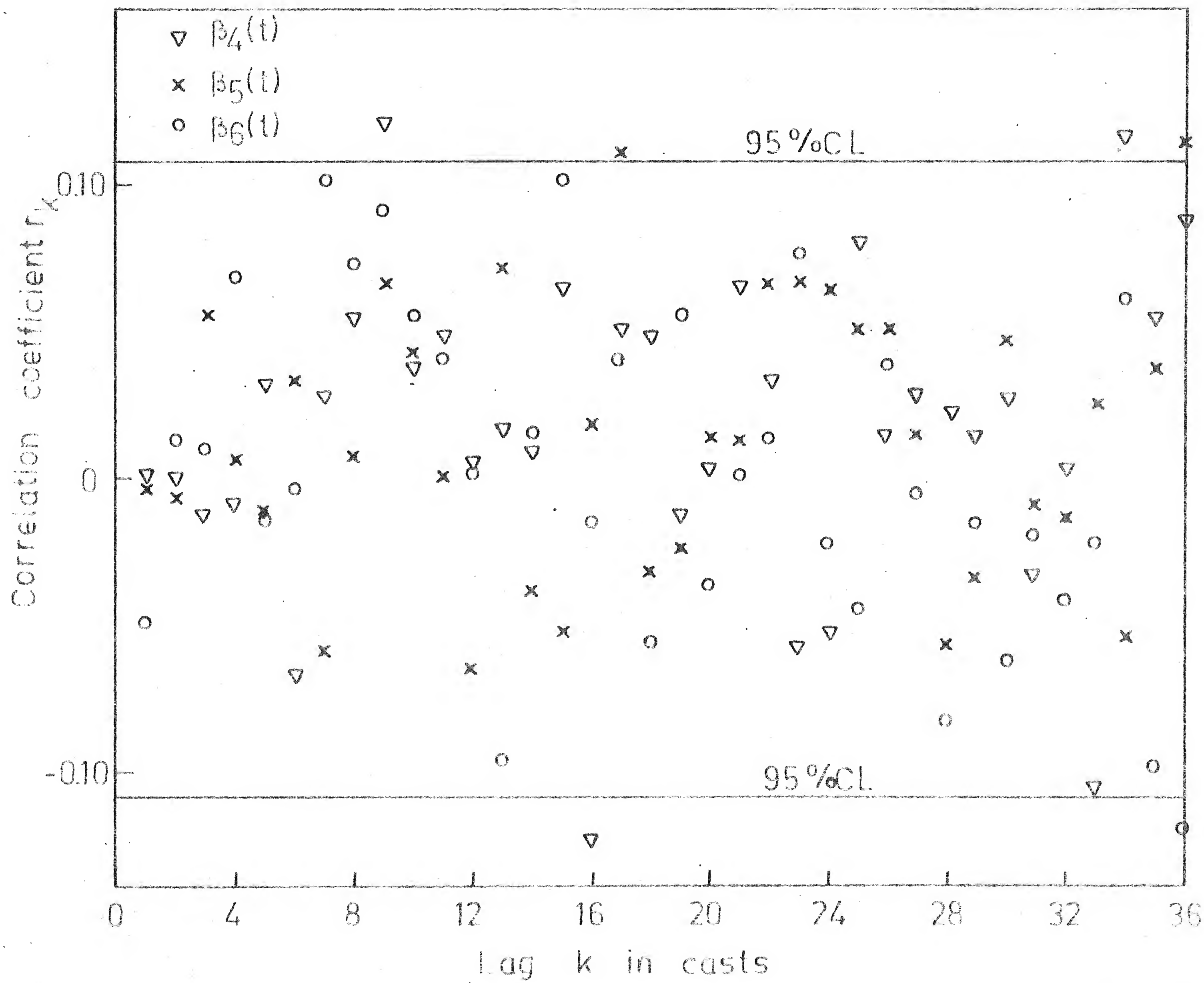


Fig. 4.2 b - Correlogram of the orthogonal vectors $\beta_4(t)$ to $\beta_6(t)$.

between all series of the vector $\underline{\beta}(t)$ **were** calculated. The cross-correlation functions have been plotted in Figures 4.3 to 4.7. These plots indicate that the series $\beta_i(t)$, i=1,...,6 are not mutually correlated. Since $\underline{\beta}(t)$ is a vector of serially and mutually uncorrelated random series, it represents the multivariate residual vector, i.e.,

$$\underline{\beta}(t) = \underline{\eta}(t) \tag{4.62}$$

The multivariate model can be written as,

$$\begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \varepsilon_3(t) \\ \varepsilon_4(t) \\ \varepsilon_5(t) \\ \varepsilon_6(t) \end{bmatrix} = \begin{bmatrix} 1.0 & & & & & \\ 0.0252 & 1.0 & & & & \\ 0.5512 & -0.0798 & 1.0 & & & \\ 0.1390 & 0.0648 & -0.0324 & 1.0 & & \\ 0.0360 & 0.1718 & -0.3021 & 0.0238 & 1.0 & \\ 0.0855 & -0.0960 & 0.0528 & -0.1190 & -0.5 & 1.0 \end{bmatrix} \times$$

$$[\eta_1(t), \eta_2(t), \eta_3(t), \eta_4(t), \eta_5(t), \eta_6(t)]^T \tag{4.63}$$

with $\hat{\sigma}^2_{\eta_1} = 0.143$, $\hat{\sigma}^2_{\eta_2} = 0.410$, $\hat{\sigma}^2_{\eta_3} = 0.953$,

$\hat{\sigma}^2_{\eta_4} = 0.815$, $\hat{\sigma}^2_{\eta_5} = 0.676$, $\hat{\sigma}^2_{\eta_6} = 0.641$

The multivariate model given by Eqn. 4.63 is also a principal component model.

4.5.2 Testing of Multivariate Residuals:

The multivariate residuals should be distributed normally and they should be serially as well as mutually independent.

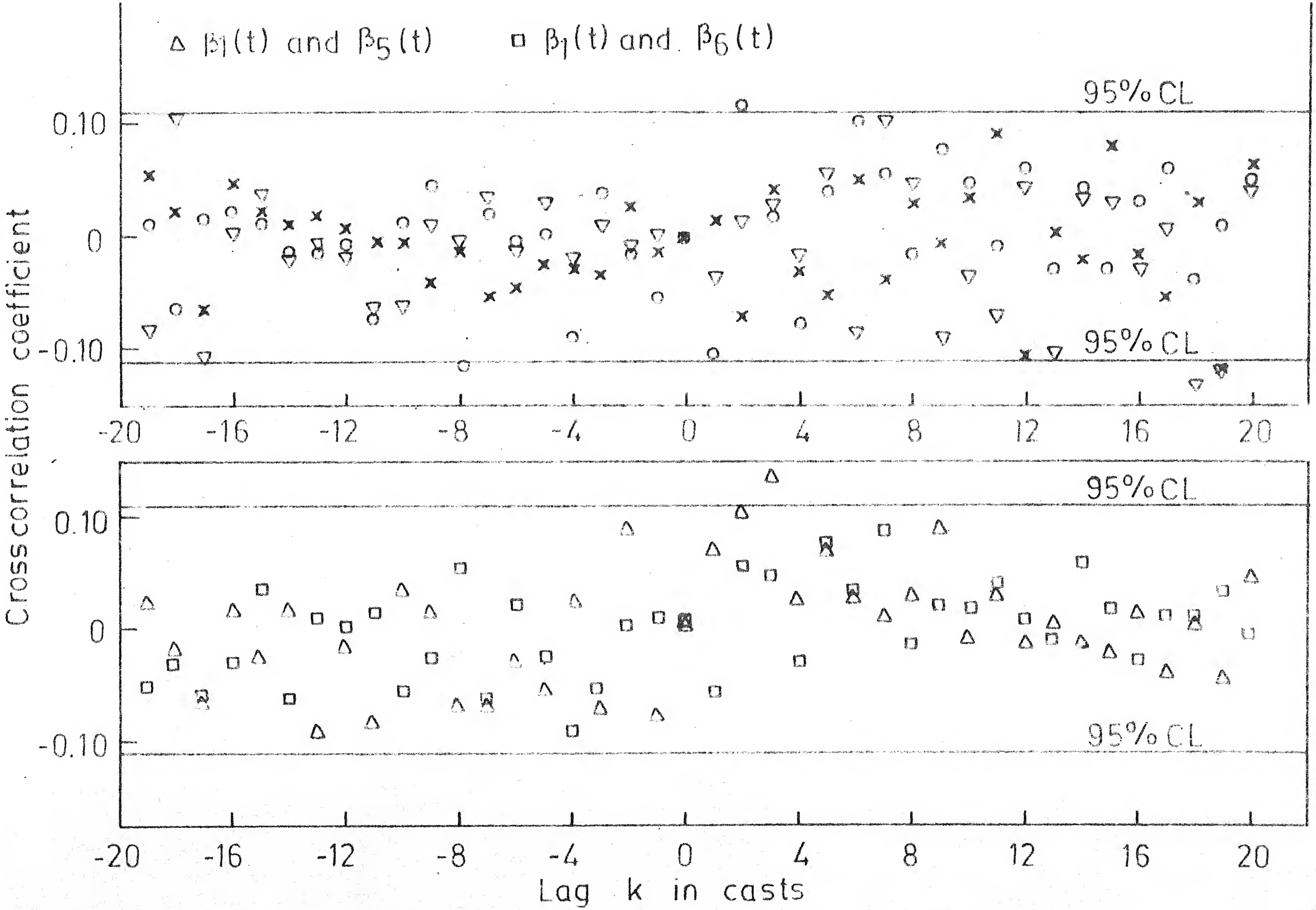


Fig. 4.3 - Cross correlogram between $\beta_1(t)$ and succeeding orthogonal vectors.

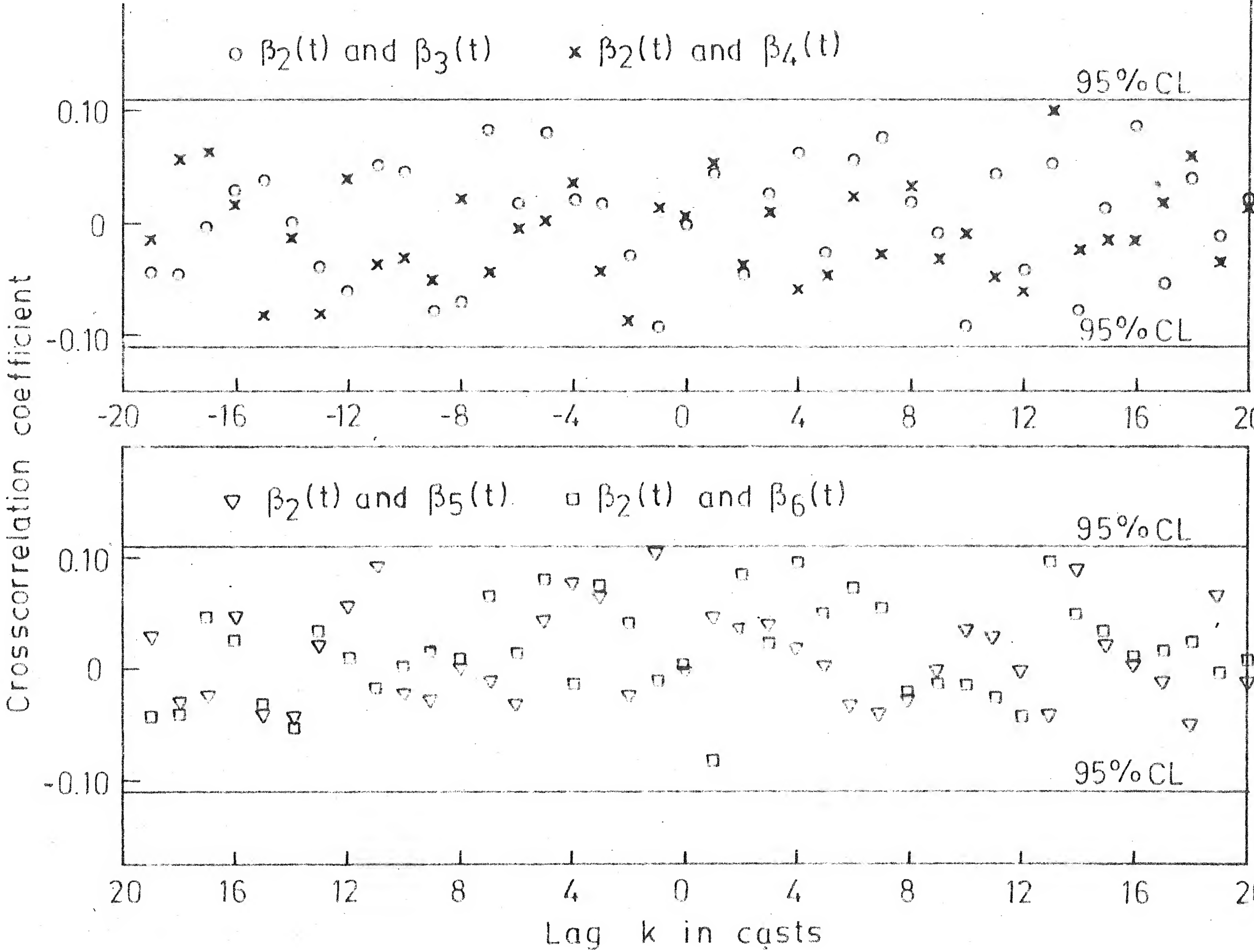


Fig. 4.4 - Cross correlogram between $\beta_2(t)$ series and succeeding orthogonal vectors.

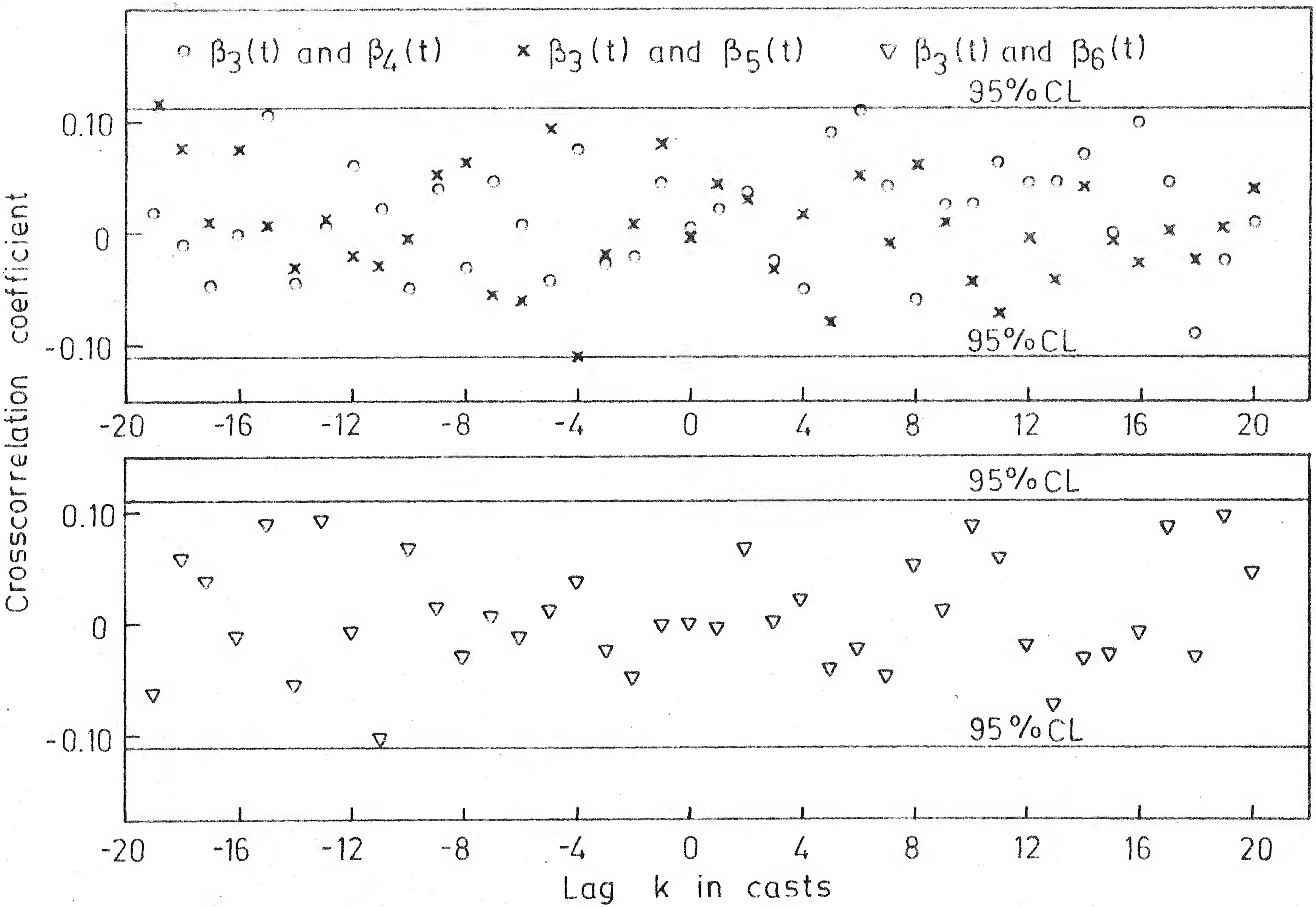


Fig. 4.5 - Cross correlogram between $\beta_3(t)$ series and succeeding orthogonal vectors.

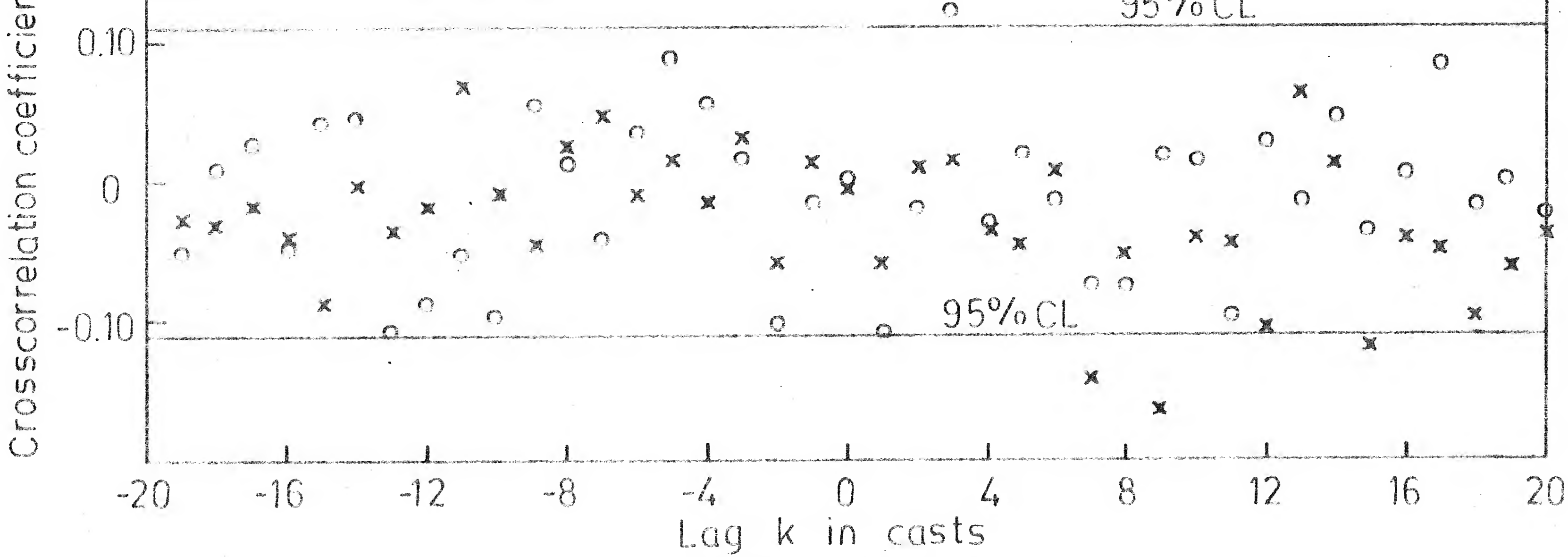


Fig. 4.6 - Cross correlogram between $\beta_4(t)$ series and succeeding orthogonal vectors.

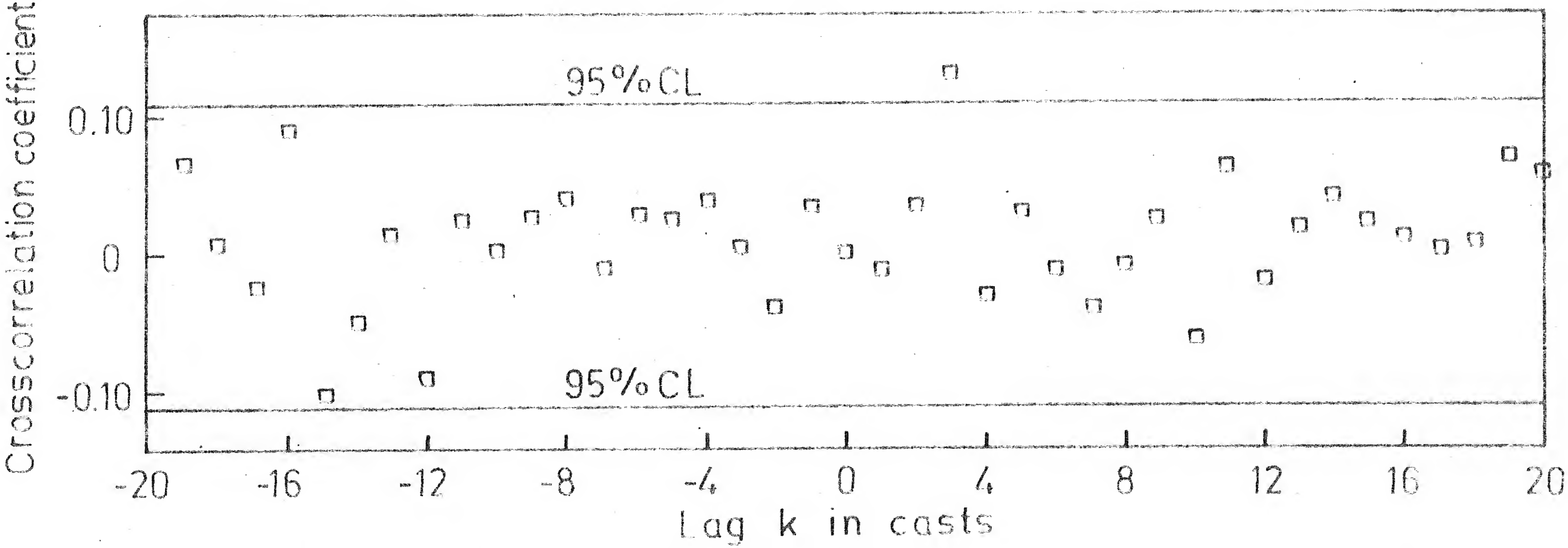


Fig. 4.7 - Cross correlogram between $\beta_5(t)$ series and succeeding orthogonal vector $\beta_6(t)$

Normality: The normality of the residuals was tested using Eqn. 3.58. The results are shown in Table 4.2. In all the cases the calculated value of chi-square statistic is less than the theoretical value and so it is concluded that all are nearly normally distributed.

Independence Within the Time Series: The serial independence of multivariate residuals is tested from correlogram of the residuals. It was found earlier that $\underline{\beta}(t) = \underline{\eta}(t)$, (Eqn. 4.62). The correlogram of $\beta_i(t)$, $i=1,2,\ldots,6$ is shown in Figures 4.2(a) and 4.2(b). They indicate that all autocorrelation functions lie within the 95 per cent confidence limit. It is hence concluded that the multivariate residuals are not significantly different from pure random series at 95 per cent confidence level.

Independence Among the Time Series: The mutual independence of the multivariate residuals is tested by cross-correlogram between each pair of residuals. The cross-correlation functions between series $\beta_i(t)$ and succeeding orthogonal vectors $\beta_j(t)$; $i = 1,\ldots,5$; $j = 1,\ldots,6$ have been shown in Figures 4.3 to 4.7. Also shown in the figures are the 95 per cent confidence limits. From these figures it is concluded that the orthogonal vectors and hence the multivariate residuals(as $\underline{\beta}(t) = \underline{\eta}(t)$) are mutually uncorrelated.

A Q-statistic similar to that given by Eqn. 3.59 but in which the autocorrelation function is replaced by cross-correlatio function is used to test the mutual independence of the series,

TABLE 4.2

TEST FOR NORMALITY OF MULTIVARIATE RESIDUALS

Series	Degrees of Freedom	Theoretical Value at 95 per cent CL	Calculated Value, Eqn. (3.58)
$\eta_1(t)$	18	28.87	27.72
$\eta_2(t)$	22	33.93	28.76
$\eta_3(t)$	28	41.34	35.65
$\eta_4(t)$	21	32.67	30.36
$\eta_5(t)$	24	36.15	25.98
$\eta_6(t)$	25	37.65	21.22

viz.,

$$Q = N \sum_{i=-K'}^{K'} \hat{r}_{\eta_k \eta_j}(i)^2 \qquad (4.64)$$

The calculated value of Q is compared with the theoretical value at 95 per cent confidence level. If the former is less than latter, the series under consideration can be considered to be mutually independent.

The chi-square test for serial and mutual independence of the multivariate residuals are given in Tables 4.3 and 4.4 respectively, and they show that the multivariate residuals are serially and mutually uncorrelated.

4.5.3 Transfer Function Model:

The details of developing transfer function model have been given in Section 4.3. As TTSMA model is also a principal component model, the vector $\underline{u}(t)$ of Eqn. 4.53 is equal to multivariate residuals vector $\underline{\eta}(t)$ and the matrix $\underline{\underline{L}}(B)$ of Eqn. 4.53 is equal to the matrix $\underline{\underline{\Psi}}^{\cap}(B)$ of TTSMA model.

Thus,

$$\underline{\underline{L}}(B) = \left[\begin{array}{ccc|ccc} 1.0 & & & & & \\ 0.0252 & 1.0 & & & & \\ 0.5512 & -0.0798 & 1.0 & & & \\ \hline 0.1391 & 0.0648 & -0.0324 & 1.0 & & \\ 0.0360 & 0.1718 & -0.3021 & 0.0238 & 1.0 & \\ 0.0855 & -0.0960 & 0.0528 & -0.1190 & -0.5000 & 1.0 \end{array}\right]$$

(4.65)

TABLE 4.3

TEST FOR SERIAL INDEPENDENCE OF THE MULTIVARIATE RESIDUALS

Series	Q - Statistic ,	Eqn. (3.59)	
$\eta_1(t)$	15.33	29.59	32.37
$\eta_2(t)$	12.43	24.87	31.51
$\eta_3(t)$	5.87	17.75	19.44
$\eta_4(t)$	9.84	23.09	26.83
$\eta_5(t)$	6.63	19.64	23.94
$\eta_6(t)$	20.04	34.75	39.80
Degrees of Freedom	12	24	30
Theoretical Value of χ^2 at 95 per cent CL	21.03	36.15	43.77

TABLE 4.4

TEST FOR MUTUAL INDEPENDENCE OF MULTIVARIATE RESIDUALS

i	j	Q-Statistic $\hat{Q} = 350 \sum_{k=-K'}^{K'} r^2_{\eta_i \eta_j}(k)$		
1	2	34.01	43.51	51.18
1	3	16.73	35.89	43.87
1	4	18.67	51.91	65.10
1	5	31.15	49.24	65.44
1	6	18.85	31.33	42.50
2	3	33.81	64.64	74.54
2	4	14.20	57.52	63.32
2	5	16.94	45.99	50.67
2	6	23.85	55.33	77.81
3	4	19.94	48.94	62.32
3	5	35.74	51.57	59.99
3	6	15.46	43.12	61.78
4	5	30.01	58.32	78.73
4	6	35.99	54.92	67.37
5	6	13.99	45.89	70.31
Degrees of freedom		24	48	60
Theoretical Value of χ^2 at 95 per cent CL		36.15	65.40	79.08

and the variance-covariance matrix of $\underline{u}(t)$ is

$$\Sigma_u = \begin{bmatrix} 0.143 & & & & & \\ & 0.410 & & & & \\ & & 0.953 & & & \underline{0} \\ & & & 0.815 & & \\ \underline{0} & & & & 0.676 & \\ & & & & & 0.641 \end{bmatrix} \quad (4.66)$$

The matrix $\underline{\underline{L}}(B)$ is then partitioned as indicated in Eqn. 4.65, thus

$$\underline{\underline{L}}_{11}(B) = \begin{bmatrix} 1.0 & & \\ 0.0252 & 1.0 & \\ 0.5512 & -0.0798 & 1.0 \end{bmatrix}$$

$$\underline{\underline{L}}_{12}(B) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{L}}_{21}(B) = \begin{bmatrix} 0.1391 & 0.0648 & -0.0324 \\ 0.0360 & 0.1718 & -0.3021 \\ 0.0855 & -0.0960 & 0.0525 \end{bmatrix} \quad (4.67)$$

$$\underline{\underline{L}}_{22}(B) = \begin{bmatrix} 1.0 & & \\ 0.0238 & 1.0 & \\ -0.1190 & -0.5 & 1.0 \end{bmatrix}$$

The variance-covariance matrix, $\Sigma_{\zeta'}$, of the white noise process $\underline{\zeta}'(t)$ is given by

$$\Sigma_{\xi'} = \begin{bmatrix} 0.815 & & \\ & 0.676 & \underline{0} \\ \underline{0} & & 0.641 \end{bmatrix} \tag{4.68}$$

The model for blast furnace noise is calculated using Eqns. 4.39 and 4.57; hence.

$$\begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{bmatrix} = \begin{bmatrix} 1.0 & & \\ 0.0238 & 1.0 & \\ -0.1190 & -0.50 & 1.0 \end{bmatrix} \begin{bmatrix} \xi'_1(t) \\ \xi'_2(t) \\ \xi'_3(t) \end{bmatrix} \tag{4.69}$$

The variance-covariance matrix $\Sigma_{\xi''}$ of the white noise process $\underline{\xi}''(t)$ is given by

$$\Sigma_{\xi''} = \begin{bmatrix} 0.143 & & \\ & 0.410 & \underline{0} \\ \underline{0} & & 0.953 \end{bmatrix} \tag{4.70}$$

The model for feedback noise is calculated using Eqns. 4.41 and 4.58; hence

$$\begin{bmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{bmatrix} = \begin{bmatrix} 1.0 & & \\ 0.0252 & 1.0 & \\ 0.5512 & -0.0798 & 1.0 \end{bmatrix} \begin{bmatrix} \xi''_1(t) \\ \xi''_2(t) \\ \xi''_3(t) \end{bmatrix} \tag{4.71}$$

The model for blast furnace dynamics is calculated using Eqn. 4.54; hence

$$\underline{\underline{V}}(B) = \begin{bmatrix} 0.1554 & 0.0622 & -0.0324 \\ 0.1987 & 0.1476 & -0.3021 \\ 0.0587 & -0.0918 & 0.0528 \end{bmatrix} \qquad (4.72$$

The feedback dynamics is calculated using Eqn. (4.55); hence

$$\underline{\underline{C}}(B) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (4.73)$$

This indicates that controls of the order of one cast interval and above are not present in the system.

The above relationships (from Eqns. 4.65 to 4.73) are in terms of prewhitened input and output variables. In order to obtain the transfer function model in terms of actual input and output variables, the multivariate model given by Eqn. 4.63 has to be first recoupled. Recoupling consists of substituting for the vector, $\underline{\varepsilon}(t)$ of univariate residuals in terms of the vector $\underline{y}(t)$ of the actual input-output variables using the univariate models.

A summary of all univariate models fitted to series A to F has been given in Table 3.3 and the models of best fit are selected in Table 3.4. Referring these Tables, we may write expressions for $\varepsilon_i(t)$, i=1,2,...,6 as follows:

$$\begin{aligned}
(1.0-0.797B + 0.197B^2 - 0.151B^3)\, y_1(t) &= \varepsilon_1(t) \\
(1.0-0.692B + 0.004B^2 - 0.121B^3)\, y_2(t) &= \varepsilon_2(t) \\
(1.0-0.684B + 0.205B^2 - 0.344B^3)\, y_3(t) &= \varepsilon_3(t) \qquad (4.74) \\
(1.0-0.237B - 0.308B^2)\, y_4(t) &= \varepsilon_4(t) \\
(1.0-0.473B)\, y_5(t) &= \varepsilon_5(t) \\
(1.0+0.222B - 0.354\, B^2) y_6(t) &= (1.0+0.582B)\varepsilon_6(t)
\end{aligned}$$

where $y_i(t)$, i=1,...,6 stand for the standardized series in the

order A to F, and in the above equation, the (3,0,0) model for the last 127 observations of blast flow rate series has been used. Equations 4.74 may also be written in matrix notation as follows:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \\ y_5(t) \\ y_6(t) \end{bmatrix} = \begin{bmatrix} (1.0-0.97B+0.197B^2-0.151B^3)^{-1} & & & & & \underline{0} \\ & (1.0-0.692B+0.004B^2-0.121B^3)^{-1} & & & & \\ & & (1.0-0.684B+0.205B^2-0.344B^3)^{-1} & & & \\ & & & (1.0-0.237B-0.308B^2)^{-1} & & \\ & & & & (1.0-0.473B)^{-1} & \\ \underline{0} & & & & & \frac{(1.0-0.582B)}{(1.0+0.222B-0.354B^2)} \end{bmatrix} \times$$

$$[\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t), \varepsilon_4(t), \varepsilon_5(t), \varepsilon_6(t)]^T \qquad (4.75)$$

or, $\underline{y}(t) = \underline{\underline{U}}(B)\ \underline{\varepsilon}(t)$ (4.76)

On combining Eqn. 4.76 with Eqn. 4.63 the relationship between actual input-output variables and the multivariate residuals is obtained as follows:

$$\underline{y}(t) = \underline{\underline{U}}(B)\ \underline{\underline{\Psi}}(B)\ \underline{n}(t) \qquad (4.77)$$

or $\underline{y}(t) = \underline{\underline{L}}'(B)\ \underline{n}(t)$ (4.78)

Equation 4.78 is similar to Eqn. 4.53 and by the method described in Section 4.3 the corresponding transfer functions can be obtained. The model for blast furnace noise for actual input-output variables is then as follows:

$$\begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{(1.0-0.237B-0.308B^2)} & & 0 \\ \frac{0.0238}{(1.0-0.473B)} & \frac{1.0}{(1.0-0.473B)} & \\ \frac{-0.1190-0.0692B}{(1.0+0.222B-0.354B^2)} & \frac{-0.50-0.291B}{(1.0+0.222B-0.354B^2)} & \frac{1.0+0.582B}{(1.0+0.222B-0.354B^2)} \end{bmatrix}$$

$$\times \, [\zeta_1'(t), \; \zeta_2'(t), \; \zeta_3'(t)]^T \qquad (4.79)$$

The variance-covariance matrix of $\underline{\zeta}'(t)$ is given by Eqn. 4.68.

The model for feedback noise for actual input-output variables is as follows:

$$\begin{bmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{bmatrix} = \begin{bmatrix} (1.0-0.97B+0.197B^2-0.151B^3)^{-1} & & 0 \\ \frac{0.0252}{(1.0-0.692B+0.004B^2-0.121B^3)} & \frac{1.0}{(1.0-0.692B+0.004B^2-0.121B^3)} & \\ \frac{0.5512}{(1.0-0.684B+0.205B^2-0.344B^3)} & \frac{-0.0798}{(1.0-0.684B+0.205B^2-0.344B^3)} & \frac{1.0}{(1.0-0.684B+0.205B^2-0.344B^3)} \end{bmatrix}$$

$$\times \, [\zeta_1''(t), \; \zeta_2''(t), \; \zeta_3''(t)]^T \qquad (4.80)$$

The variance covariance matrix of $\underline{\zeta}''(t)$ is given by Eqn. 4.70. The model for feedback dynamics is given by Eqn. 4.73 as before indicating absence of feed back control of order of one cast interval or larger.

The elements $V_{ij}(B)$, i = 1 to 3 and j=1 to 3, of the blast furnace dynamics matrix $\underline{\underline{V}}(B)$ are as follows:

$$V_{11}(B) = V_{AD}(B) = \frac{0.1554 - 0.1509B + 0.02963B^2 - 0.0231B^3}{1.0 - 0.237B - 0.308B^2}$$

$$V_{12}(B) = V_{BD}(B) = \frac{0.0622 - 0.0431B + 0.00026B^2 - 0.00755B^3}{1.0 - 0.237B - 0.308B^2}$$

$$V_{13}(B) = V_{CD}(B) = \frac{-0.0324 + 0.0222B - 0.0066B^2 + 0.0111B^3}{1.0 - 0.237B - 0.308B^2}$$

$$V_{21}(B) = V_{AE}(B) = \frac{0.1987 - 0.1937B + 0.0391B^2 - 0.03B^3}{1.0 - 0.473B}$$

$$V_{22}(B) = V_{BE}(B) = \frac{0.1477 + 0.01023B + 0.0006B^2 - 0.0179B^3}{1.0 - 0.473B}$$

$$V_{23}(B) = V_{CE}(B) = \frac{-0.3021 + 0.206B - 0.062B^2 + 0.1040B^3}{1.0 - 0.473B}$$

$$V_{31}(B) = V_{AF}(B) = \frac{0.05872 - 0.02162B - 0.02150B^2 - 0.00223B^3 - 0.0052B^4}{1.0 + 0.222B - 0.354B^2}$$

$$V_{32}(B) = V_{BF}(B) = \frac{-0.0918 + 0.00993B + 0.0366B^2 + 0.0108B^3 + 0.0065B^4}{1.0 + 0.222B - 0.354B^2}$$

$$V_{33}(B) = V_{CF}(B) = \frac{0.0528 - 0.0045B - 0.0101B^2 - 0.0119B^3 - 0.0106B^4}{1.0 + 0.222B - 0.354B^2}$$

In the above expressions $V_{AD}(B)$ represents the transfer function between sinter-to-coke ratio (Series A) and hot metal temperature (Series D) and so on. On simplifying the above expressions we get

$$V_{AD}(B) = 0.075B - 0.1545 + \frac{0.344}{(0.45B+1.0)} + \frac{0.0363}{(0.687B-1.0)}$$

$$V_{BD}(B) = 0.0234B - 0.0186 + \frac{0.0925}{(0.45B+1.0)} + \frac{0.0116}{(0.687B-1.0)}$$

$$V_{CD}(B) = -0.036B + 0.0456 - \frac{0.099}{(0.45B+1.0)} + \frac{0.00137}{(0.687B-1.0)}$$

$$V_{AE}(B) = 0.0635B^2 + 0.0516B + 0.518 + \frac{0.3193}{(0.473B - 1.0)}$$

$$V_{BE}(B) = 0.0378B^2 + 0.0787B + 0.144 - \frac{0.0037}{(0.473B-1.0)}$$

$$V_{CE}(B) = -0.22B^2-0.334B-1.14 - \frac{0.838}{(0.473B-1.0)}$$

$$V_{AF}(B) = 0.0147B^2 + 0.01545B + 0.112 + \frac{0.0727}{(0.495B-1.0)} + \frac{0.019}{(0.716B+1.0)}$$

$$V_{BF}(B) = -0.01825B^2-0.0421B - 0.182 - \frac{0.0785}{(0.495B-1.0)} + \frac{0.0118}{(0.716B+1.0)}$$

$$V_{CF}(B) = 0.03B^2 + 0.0525B - 0.1480 + \frac{0.113}{(0.495B-1.0)} + \frac{0.018}{(0.716B+1.0)}$$

The transfer function between the input and output given by Eqn. 4.38a assumes a pure delay of order b_{ij} and a time constant represented by a number of linear difference operators. The above equations indicate that the transfer function can be considered as weighted sum of a number of delay operators and time constants. It is known that all discrete models with finite inputs are stable and a comparison with differential system indicates that the forward and backward approximations to differential system should be so chosen that they are stable. Hence, we may consider the above representation to consist of

terms involving shift operator B which represent the time delay and the other terms which represent the time constant. For example, the transfer function between the input A and output F consists of the following:

(i) a dead time of 2 cast intervals with a weightage of 0.0147

(ii) a dead time of 1 cast interval with a weightage of 0.01545

(iii) a weightage of 0.112 with no delay

(iv) a weightage of 0.0727 with a time constant T_{c_1} of 0.495 cast interval, and

(v) a weightage of 0.0116 with a time constant T_{c_2} of 0.716 cast interval.

The total weightage W, on the input is hence 0.2338 and the relative weights for the operators W_1 to W_5 are, respectively 0.063, 0.0661, 0.479, 0.311, and 0.0814. Hence, the time constant and time delay for the system are given by

$$T_c = T_{c_1} W_4 + T_{c_2} W_5$$

and

$$T_D = 2W_1 + W_2$$

A block diagram representation of the input-output relationship is given in Figure 4.8. The total weight on the input and the fractional weights as well as the time constants and dead times have been calculated for all input-output pairs and are shown in Table 4.5.

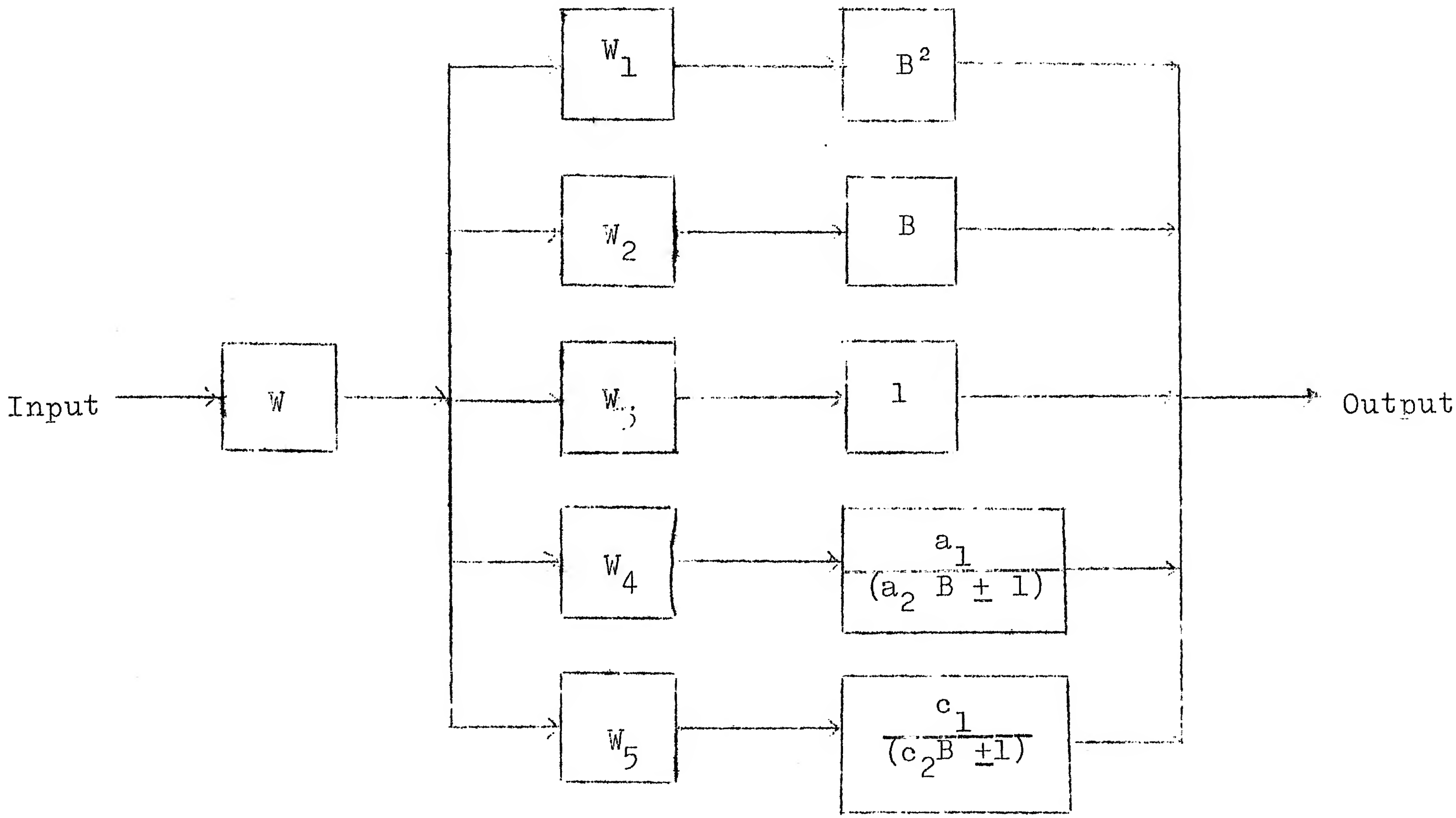


FIG.4.8 BLOCK DIAGRAM REPRESENTATION OF INPUT-OUTPUT RELATIONSHIP

TABLE 4.5

PARAMETER ESTIMATES OF TRANSFER FUNCTION MODEL

Transfer function	Total weight, W	Weightage for B^2	B	1	T_{c_1}	T_{c_2}	Total Time constant, T_c	Dead Time, T_D
$V_{AD}(B)$	0.6098	-	0.1230	0.2535	0.5640	0.0595	0.2950	0.1230
$V_{BD}(B)$	0.1469	-	0.1590	0.1260	0.6250	0.0925	0.3445	0.1590
$V_{CD}(B)$	0.1816	-	0.1983	0.2490	0.5450	0.0077	0.2470	0.1983
$V_{AB}(B)$	0.9524	0.0667	0.0542	0.5440	0.3350	-	0.1590	0.1876
$V_{BE}(B)$	0.2642	0.1430	0.2980	0.5450	0.0140	-	0.0066	0.5840
$V_{CE}(B)$	2.5320	0.0880	0.1320	0.4500	0.3310	-	0.1570	0.3080
$V_{AF}(B)$	0.2338	0.0630	0.0661	0.4790	0.3110	0.0814	0.2122	0.1921
$V_{BF}(B)$	0.3327	0.0550	0.1202	0.5470	0.2360	0.0355	0.1173	0.2362
$V_{CF}(B)$	0.3617	0.0830	0.1450	0.4100	0.3120	0.0504	0.1790	0.3110

Some workers have derived the transfer functions for the input-output pairs, blast temperature-silicon content and blast humidity-silicon content. These have been shown in Table 4.6. It can be seen from this table that, in general, a time constant of about 6 hours exists between blast temperature and silicon content. The response of silicon content to blast humidity is very fast, the time constant is of the order of two hours, which is less than a cast interval. In the present study, blast temperature was not taken into account because when data were collected, it was kept at a constant level of 1000°C. Castore et al. [62] have shown that a dead time of 0.75 times cast interval and a time constant of 0.75 times cast interval exist between ore-to-coke ratio and silicon content of hot metal.

For the present study, the data on output variables were available only at cast intervals but the data on input variables were available at one hour time interval. Hence, the input variables were averaged between successive casts as a result of which a time delay of the order of half cast interval already existed between inputs and outputs.

It can be seen from Table 4.5 that the total lag between the average input in a cast interval and output at the end of cast is of the order of 0.35 to 0.59 casts. Hence, the actual lag between input and output is of the order of 0.85 to 1.09 casts. This is comparable to the results obtained by other workers.

TABLE 4.6

TRANSFER FUNCTION MODELS FOR BLAST FURNACE

Worker	Blast humidity-silicon content of hot metal	Blast temperature-silicon content of hot metal
Wood[40]	$\frac{-0.0580}{1+2.8s}$	$\frac{0.0052}{1+6s}$
Barbieri [see 40]	$\frac{-0.053}{1+3.4s}$	$\frac{0.0025}{1+6.2s}$
Staib et al. [see 40]	$\frac{-0.023}{1+2s}$	$\frac{0.002}{1+14s}$
Castore et al. [62]	$\frac{-0.8}{1+1.6s}$	$\frac{0.25}{1+3s}$
Vidal et al. [58]	$\frac{-0.021}{1+s} + \frac{0.007\ e^{-8s}}{(1+1.5s)}$	$\frac{0.004}{1+6s}$
Present study	$0.0378B^2 \mp 0.0787B + 0.144 + \frac{-0.0037}{(0.473B-1)}$	-

Results from Table 4.6 indicate the time constant of the order of 1.6 to 3.5 hours for blast humidity - silicon content transfer function. Since the cast interval is of the order of 2.5 to 3.0 hours, 1.09 cast intervals correspond to around 2.7 to 3.3 hours and this is in agreement with the earlier results. Vidal's result indicates a time delay of 8 hours on 25 per cent of the input corresponding to an average time delay of about 2 hours. This may be compared with the results of this study, namely, 14.3 per cent weight on two time delays and 29.8 per cent weight on one time delay corresponding to an overall time delay of 0.584 cast interval or 1.5 to 1.8 hours. Since the multivariate model gives greater details of the transformation and gives comparable results to one of the input-output pairs, the other results may be considered to be reasonable in the absence of information to the contrary.

Our results indicate that the effect of random component of input on random component of output does not last for a cast interval or more. This is indicated by Eqn. 4.72, for blast furnace dynamics which does not contain any term involving the operator B. This is of same order of magnitude as in the case of standardized input-output variables. However, the latter indicates the details of transformation and particularly the fact that the transformations involved delays on a part of the input, zero delay on another part and transformations on the remainder due to time constants. Thus, for example, the outputs

silicon and sulphur contents of pig iron are influenced directly by the characteristics of the inputs in not only the cast interval just prior to that but also those of two earlier casts. The effect of inputs on hot metal temperature persists only in the current and next cast intervals. These results seem to be interesting. The order of significance of the inputs in the earlier cast intervals are also indicated in Table 4.5 and they are generally of the order of 12 to 35 per cent.

The feedback dynamics matrix, in both the cases, viz., in terms of prewhitened input-output variables and in terms of actual input-output variables was found to be zero. This indicates that there is no feedback of the order of one cast interval or more from any of the output variables to the input variables.

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CHAPTER 5

SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

5.1 SUMMARY AND CONCLUSIONS:

In the present study mathematical models for blast furnace process have been reviewed. Mathematical models have been broadly classified as deterministic and probabilistic; the deterministic models were further subclassified as steady state and dynamic models, and the probabilistic models were subdivided into regression models, linear process models and time series models.

A decoupled multivariate time series model has been developed for the blast furnace No.1 of Bokaro Steel Limited, Bokaro Steel City, using data on three input variables, namely, sinter-to-coke ratio, blast humidity and blast flow rate; and three output variables, namely, temperature of hot metal and its silicon and sulphur content.

All the variables,but blast flow rate, were found to be stationary. The blast flow rate series had two jumps and hence it was divided into 3 parts,each of which was stationary. All the variables were found to be free from cyclicity. In order to remove persistence (internal and/or external dependence) a univariate model was fitted to each of the series. The resulting residuals which are normally distributed and serially independent are called as 'prewhitened series'.

A multivariate model was then fitted to the prewhitened series in terms of serially and mutually uncorrelated random components. The multivariate model is referenced to as a decoupled model since it decouples the within-the-series variability in terms of univariate models from the among-the-series variability in terms of the multivariate model. The advantages of decoupling are (i) as the parameters of univariate and multivariate models are estimated separately, their number is comparable to those of univariate and multivariate models and difficulties involved in simultaneously estimating all parameters are avoided; and (ii) the most appropriate form and order of model can be selected independently for each variable and hence there is greater flexibility in the choice of the models.

The method proposed for multivariate modelling is the generalization of the method developed earlier by Phadke et al. It uses Gram-Schmidt procedure to obtain orthogonal vectors. A principal component model was derived in terms of prewhitened series and orthogonal vectors. A multivariate model was then developed by calculating the cross correlation functions between orthogonal vectors and multiple regression analysis. It was found that all orthogonal vectors form a family of serially and mutually uncorrelated random components. Hence, the multivariate model was also a principal component model.

From the multivariate model, transfer function model between prewhitened input-output variables was developed. In

order to obtain the relationship between actual input-output variables the multivariate model was recoupled and then transfer function model was derived. It was found that the relationships between inputs and outputs consist of terms involving shift operator B which represents the time delay and other terms representing the time constants. The order of significance of the inputs in the earlier cast intervals was found to be 12 to 35 per cent. The average time lag between input and output was found to be varying from 0.85 to 1.09 cast intervals. These results generally agree with the results proposed earlier by other workers.

Feedback dynamics was found to be absent. This indicates that the control action has its affect atmost within one cast interval and there is no effect beyond that.

5.2 SUGGESTIONS FOR FURTHER STUDY:

In order to determine more accurately time constants and time delays, data are to be used at time intervals much less than one cast interval. Data on input variables were available at one hour interval. In this study they were averaged over the cast interval. For better results they should be sampled and used at shorter time intervals. Data on output variables, however, are generally available only at the time of cast.

It is suggested that to notice the effect of operating variables on the pig iron process, one has to consider the extent of direct and indirect reduction processes which are

functions of ore properties; its reactivity and the flow pattern of the gases and solids in the furnace. Therefore one of the important indicating variables about the state of the process may be 'excess heat' above the normal requirement. This could be measured either in terms of top gas analysis (CO/CO_2 ratio, H_2/H_2O ratio) or material and energy balance over different sections of the blast furnace.

Furthermore there is a good correlation between the variation of the silicon content of molten metal with that of silica in the burden. Using these and other pertinent variables, more comprehensive models for system dynamics of the blast furnace process may be developed using the decoupled multivariate modelling approach. They may then be used in developing control models and control systems.

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APPENDIX A

MARQUARDT ALGORITHM FOR NON-LINEAR LEAST SQUARE ESTIMATION

Let $\underset{\sim}{\xi} = (\xi_1, \xi_2, \ldots\ldots \xi_K)$ denote the parameters of the model, that is $\underset{\sim}{\xi} = (\varphi_1, \varphi_2, \ldots, \varphi_p; \theta_1, \theta_2, \ldots\ldots, \theta_q)$ where $K = p+q$. To start with the initial approximation $\underset{\sim}{\xi}_o$, a convergence parameter ε and the parameters π and F_2 are specified which constrain the search. During the search, the values $\varepsilon(t)$ and the derivative

$$x_{i,t} = - \frac{\partial \varepsilon(t)}{\partial \xi_i}$$

are evaluated, numerically, at each stage of iteration. The derivatives are obtained from

$$x_{i,t} = [\varepsilon(t, \xi_{1,0}, \ldots, \xi_{i,0}, \ldots\ldots, \xi_{K,0})$$

$$-\varepsilon(t, \xi_{1,0}, \ldots, \xi_{i,0} + \delta_i, \ldots\ldots, \xi_{K,0})] / \delta_i$$

Stage 1: With $\varepsilon(t)$ and $x_{i,t}$ supplied from the current parameter values the following quantities are formed:

1. The matrix

$$\underset{=}{A} = [A_{ij}]$$

where

$$A_{ij} = \sum_{t=Q}^{N} x_{i,t}\, x_{j.t}$$

2. The vector $\underset{\sim}{g}$ with elements $g_1, g_2, \ldots\ldots$

where

$$g_i = \sum_{t=Q}^{N} x_{i,t}\, \varepsilon(t)$$

3. The scaling quantities

$$D_i = \sqrt{A_{ii}}$$

Stage 2: The modified (scaled and constrained) linearized equations

$$\underline{\underline{A}}^* \quad \underline{h}^* = \underline{g}^*$$

are constructed according to

$$A^*_{ij} = A_{ij}/D_i D_j \qquad i \neq j$$

$$A^*_{ii} = 1 + \pi$$

$$g^*_i = g_i/D_i$$

The equations are solved for $\underline{h}^*$ which is scaled back to give parameter correction h_j, where

$$h_j = h^*_j/D_j$$

The parameter values are constructed from

$$\underline{\xi} = \underline{\xi}_o + \underline{h}$$

and the sum of squares of residuals is calculated using,

$$S(\underline{\xi}) = [\underline{\varepsilon}(t)] \; [\underline{\varepsilon}(t)]^T$$

Stage 3: (i) If $S(\underline{\xi}) < S(\xi_o)$, the parameter corrections $\underline{h}$ are tested. If all are smaller than ε, convergence is assumed and the matrix $\underline{\underline{A}}^{-1}$ is used to calculate the variance-covariance matrix; otherwise $\underline{\xi}_o$ is reset to the value $\underline{\xi}$, π is reduced by a factor F_2 and computation returns to Stage 1.

(ii) If $S(\underset{\sim}{\xi}) > S(\underset{\sim}{\xi}_o)$, the constraint parameter π is increased by a factor F_2, and computation resumed at Stage 2. An upper bound is placed on π, and if this bound is exceeded, the search is terminated.

When convergence has occurred the residual variance and the variance-covariance matrix of estimates are calculated.

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APPENDIX B

GRAM-SCHMIDT ORTHOGONALIZATION PROCESS

The process generates a set of K orthogonal vectors $\beta_k(t)$, k=1,....,K from a set of K linearly independent vectors $\varepsilon_k(t)$, k=1,....,K by forming linear combination of $\varepsilon_k(t)$. Let $\alpha_k(t)$, k=1,....,K be a set of orthonormal vectors, then the orthonormal set $\alpha(t)$ has a property that

$$(\alpha_j(t)\cdot\alpha_k(t)) = 1 \qquad j = k$$
$$= 0 \qquad j \neq k$$

the paranthesis $(\cdot)$ means the inner product of vectors $\alpha_j(t)$ and $\alpha_k(t)$,

$$(\alpha_j(t)\cdot\alpha_k(t)) = \sum_{i=1}^{N} \alpha_j(i)\cdot\ \alpha_k(i)$$

To start with Gram-Schmidt orthogonalization process, set

$$\beta_1(t) = \varepsilon_1(t)$$

$$||\beta_1(t)|| = (\beta_1(t)\cdot\beta_1(t))^{\frac{1}{2}}$$

$$\alpha_1(t) = \frac{\beta_1(t)}{||\beta_1(t)||}$$

The second vector $\beta_2(t)$ in the set of unnormalized orthogonal vectors is found by a linear combination of $\alpha_1(t)$ and $\varepsilon_2(t)$, that is,

$$\beta_2(t) = \varepsilon_2(t) - \delta_{12}\,\alpha_1(t)$$

where δ_{12} is a constant.

Forming the inner product with $\alpha_1(t)$ we find that

$$(\beta_2(t)\cdot\alpha_1(t)) = (\varepsilon_2(t)\cdot\alpha_1(t)) - \delta_{12}(\alpha_1(t)\cdot\alpha_1(t))$$

Since $\beta_2(t)$ and $\beta_1(t)$ are required to be orthogonal it is necessary that

$$(\beta_2(t) \cdot \alpha_1(t)) = 0$$

By orthonormal property

$$(\alpha_1(t) \cdot \alpha_1(t)) = 1.0$$

Hence

$$\delta_{12} = (\varepsilon_2(t)\cdot\alpha_1(t))$$

The third vector $\beta_3(t)$ in the set of unnormalized orthogonal vectors is formed as a linear combination of $\alpha_1(t)$, $\alpha_2(t)$ and $\alpha_3(t)$ expressed as

$$\beta_3(t) = \varepsilon_3(t) - \delta_{13}\,\alpha_1(t) - \delta_{23}\,\alpha_2(t)$$

By taking inner products

$$(\beta_3(t)\cdot\alpha_1(t)) = (\varepsilon_3(t)\cdot\alpha_1(t)) - \delta_{13}(\alpha_1(t)\cdot\alpha_1(t)) - \delta_{23}(\alpha_2(t)\cdot\alpha_1(t))$$

$$(\beta_3(t)\cdot\alpha_2(t)) = (\varepsilon_3(t)\cdot\alpha_2(t)) - \delta_{13}(\alpha_1(t)\cdot\alpha_2(t)) - \delta_{23}(\alpha_2(t)\cdot\alpha_2(t))$$

Since $\beta_3(t)$ is required to be orthogonal to $\beta_1(t)$ and $\beta_2(t)$ we have

$$(\beta_3(t)\cdot\alpha_1(t)) = 0 \quad \text{and} \quad (\beta_3(t)\cdot\alpha_2(t)) = 0$$

By orthonormal property

$$(\alpha_2(t)\cdot\alpha_2(t)) = 1 \qquad (\alpha_1(t)\cdot\alpha_1(t)) = 1$$

and $(\alpha_1(t)\cdot\alpha_2(t)) = 0$

Hence

$$\delta_{13} = (\alpha_1(t)\cdot\varepsilon_3(t))$$

$$\delta_{23} = (\alpha_2(t)\cdot\varepsilon_3(t))$$

The general expression for calculating $\beta_j(t)$ from $\varepsilon_j(t)$ becomes

$$\beta_j(t) = \varepsilon_j(t) - \sum_{i=1}^{j-1} (\alpha_i(t)\cdot\varepsilon_j(t))\ \alpha_i(t)$$

or $$\beta_j(t) = \varepsilon_j(t) - \sum_{i=1}^{j-1} \delta_{ij}\ \alpha_i(t)$$

where $$\alpha_i(t) = \frac{\beta_i(t)}{\|\beta_i(t)\|} = \frac{\beta_i(t)}{(\beta_i(t)\cdot\beta_i(t))^{\frac{1}{2}}}$$

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APPENDIX C

TABLE OF DATA .
DATA HAS BEEN COLLECTED ON BLAST FURNACE AT BOKARD STEEL LIMITED.
READ HORIZONTALLY

SERIES 1 .. SINTER-TO-COKE RATIO.

2.375	2.340	2.375	2.375	2.375	2.375	2.280	2.360	2.360	2.33
2.310	2.250	2.260	2.360	2.360	2.360	2.360	2.360	2.360	2.35
2.200	2.200	2.360	2.380	2.380	2.365	2.380	2.378	2.410	2.41
2.410	2.415	2.460	2.462	2.462	2.462	2.432	2.443	2.470	2.47
2.475	2.440	2.458	2.460	2.338	2.450	2.460	2.440	2.476	2.48
2.490	2.490	2.480	2.470	2.470	2.473	2.480	2.470	2.465	2.45
2.500	2.500	2.500	2.500	2.475	2.490	2.510	2.530	2.540	2.55
2.550	2.550	2.550	2.550	2.550	2.550	2.540	2.550	2.550	2.56
2.615	2.620	2.620	2.620	2.610	2.570	2.570	2.570	2.580	2.58
2.570	2.560	2.560	2.568	2.580	2.580	2.580	2.555	2.480	2.52
2.520	2.525	2.490	2.490	2.485	2.480	2.480	2.480	2.480	2.45
2.440	2.440	2.440	2.440	2.440	2.442	2.445	2.445	2.465	2.49
2.490	2.490	2.490	2.490	2.490	2.490	2.493	2.513	2.520	2.49
2.445	2.445	2.445	2.445	2.445	2.445	2.445	2.445	2.445	2.44
2.427	2.427	2.430	2.440	2.440	2.440	2.440	2.440	2.450	2.45
2.500	2.500	2.535	2.550	2.550	2.520	2.500	2.500	2.500	2.50
2.430	2.416	2.385	2.370	2.416	2.416	2.276	2.275	2.285	2.31
2.317	2.320	2.320	2.340	2.420	2.420	2.420	2.440	2.410	2.44
2.443	2.450	2.453	2.453	2.453	2.453	2.469	2.472	2.472	2.47
2.472	2.472	2.472	2.472	2.420	2.425	2.425	2.425	2.425	2.43
2.444	2.444	2.428	2.425	2.425	2.425	2.425	2.425	2.425	2.42
2.425	2.425	2.472	2.472	2.472	2.472	2.472	2.472	2.475	2.52
2.520	2.520	2.520	2.520	2.520	2.520	2.520	2.520	2.520	2.52
2.520	2.530	2.510	2.480	2.480	2.480	2.480	2.480	2.480	2.48
2.490	2.490	2.490	2.460	2.445	2.475	2.490	2.470	2.445	2.44
2.445	2.445	2.445	2.445	2.425	2.400	2.400	2.397	2.410	2.43
2.398	2.400	2.400	2.400	2.400	2.400	2.400	2.400	2.400	2.40
2.400	2.400	2.400	2.400	2.420	2.420	2.420	2.420	2.400	2.40
2.410	2.410	2.409	2.390	2.390	2.386	2.390	2.430	2.435	2.43
2.435	2.405	2.390	2.420	2.420	2.400	2.408	2.436	2.497	2.51
2.520	2.520	2.520	2.520	2.520	2.520	2.520	2.350	2.310	2.51
2.310	2.330	2.350	2.364	2.370	2.480	2.470	2.470	2.520	2.52
2.520	2.520	2.520	2.520	2.510	2.540	2.570	2.570	2.560	2.54
2.540	2.540	2.520	2.560	2.560	2.510	2.510	2.510	2.560	2.56
2.560	2.560	2.560	2.570	2.600	2.580	2.540	2.520	2.520	2.52

SERIES 2 .. BLAST HUMIDITY .

45.0	45.3	47.7	48.0	47.7	47.0	49.5	49.3	46.3	44.0
45.67	46.3	46.5	46.3	45.3	45.0	49.62	45.0	45.0	41.67
41.33	41.0	41.67	41.0	48.67	50.0	48.5	45.67	46.0	48.0
49.0	52.0	52.5	48.67	46.0	46.5	50.0	48.5	49.67	49.0
50.5	50.0	45.0	59.0	52.0	52.0	50.0	48.0	51.3	50.3
48.0	45.5	43.0	48.67	49.0	51.0	48.67	47.67	49.0	44.0
42.0	43.0	45.67	46.0	41.5	45.0	43.3	41.5	46.3	45.67
42.0	44.67	43.3	42.5	48.0	47.33	46.0	45.67	44.0	45.0
45.67	45.67	39.33	45.5	43.0	43.0	38.67	42.5	43.0	44.5
41.0	41.0	43.0	44.0	42.0	41.0	40.0	45.0	45.3	44.5
43.3	45.5	42.5	40.67	42.5	43.0	42.0	44.0	45.0	45.0
45.0	48.0	49.0	55.33	48.0	55.0	53.0	46.0	46.0	48.0
48.0	49.5	51.5	53.0	53.5	53.0	51.0	47.0	45.67	47.0
48.33	49.0	49.67	46.8	45.67	48.0	48.0	49.0	49.0	49.0
49.0	45.3	45.5	48.0	46.5	47.0	44.0	45.0	43.0	43.0
44.0	44.0	45.67	42.5	40.67	41.0	40.0	39.0	38.0	39.0
40.0	37.3	40.0	42.0	44.0	45.67	46.0	47.0	45.67	43.5
42.0	44.0	43.3	43.0	43.5	41.3	38.0	38.0	38.33	44.67
43.0	41.0	43.0	44.33	45.0	44.0	45.0	48.0	46.0	45.5
45.0	44.67	43.0	41.0	41.0	43.33	44.5	44.0	45.0	45.0
46.0	44.33	43.0	43.0	43.33	44.0	44.0	39.0	44.0	47.0
45.0	45.0	45.0	44.0	45.0	43.67	45.0	45.0	46.0	45.0
46.0	44.67	42.0	42.5	43.0	44.0	43.0	42.0	40.0	39.5
38.67	37.0	38.0	43.0	41.0	45.3	48.0	46.67	46.0	48.0
48.0	48.0	49.0	45.0	51.33	52.0	47.5	43.67	46.0	44.0
41.0	40.0	41.0	45.5	45.0	42.33	45.5	44.0	43.0	40.33
40.0	38.67	35.0	40.33	40.0	43.33	44.0	45.0	44.0	42.0
41.33	42.0	42.0	42.0	42.0	42.0	42.5	41.33	47.0	47.0
44.33	44.67	44.67	48.5	45.67	43.0	44.67	47.33	48.0	46.67
43.0	43.5	43.33	37.33	44.0	47.67	49.33	45.0	44.0	42.33
42.0	42.0	42.8	41.67	41.67	45.0	54.0	50.0	50.0	52.0
53.0	50.0	51.0	50.0	49.0	46.67	50.0	50.0	49.33	48.0
53.67	56.0	52.0	50.0	54.33	55.0	48.5	52.0	53.0	48.0
45.33	48.0	49.0	44.0	42.67	43.33	45.0	46.67	53.5	53.0
53.0	53.0	53.0	53.5	46.0	45.0	47.0	48.5	51.0	ED48.5

SERIES 3 .. BLAST FLOW RAYE .

2950.0	2933.0	2917.5	2967.0	2983.0	2900.0	2850.0	3000.0	3100.0	3075.0
3050.0	3050.0	3050.0	3050.0	3050.0	3050.0	3050.0	3066.0	3050.0	3050.0
2750.0	2917.0	3000.0	3050.0	2950.0	3050.0	2825.0	3050.0	3150.0	3200.0
3225.0	3250.0	3400.0	3400.0	3400.0	3400.0	3300.0	3375.0	3250.0	3400.0
3400.0	3417.0	3425.0	2833.0	2167.0	3375.0	3450.0	3500.0	3500.0	3467.0
3500.0	3500.0	3500.0	3500.0	2917.0	2425.0	3083.0	3450.0	3433.0	3500.0
3500.0	3500.0	3475.0	3233.0	2900.0	3425.0	3500.0	3500.0	3550.0	3117.0
3100.0	3500.0	3500.0	3500.0	3225.0	2600.0	3383.0	3500.0	3500.0	3500.0

3500.0	3433.0	3433.0	3000.0	3117.0	3250.0	3267.0	2350.0	2925.0	3150.0
3475.0	3533.0	3600.0	3550.0	3183.0	3650.0	3550.0	3600.0	3550.0	3500.0
3500.0	3433.0	3500.0	3500.0	3350.0	3425.0	3367.0	3550.0	3550.0	3500.0
3500.0	3500.0	3500.0	3500.0	3433.0	3500.0	3500.0	3450.0	3550.0	3600.0
3600.0	3433.0	3500.0	3400.0	3500.0	3475.0	3550.0	3467.0	3450.0	3483.0
3600.0	3617.0	3475.0	2133.0	2190.0	3033.0	3483.0	3500.0	3100.0	3567.0
3600.0	3550.0	3533.0	3550.0	3583.0	3600.0	3517.0	3500.0	3600.0	3600.0
3600.0	3600.0	3600.0	3567.0	3575.0	3500.0	3500.0	3500.0	3475.0	3550.0
3625.0	3600.0	3500.0	3500.0	3500.0	3500.0	3583.0	3550.0	3475.0	3500.0
3525.0	3550.0	3700.0	3600.0	3600.0	3550.0	3533.0	3500.0	3500.0	3500.0
3500.0	3500.0	2650.0	3200.0	3500.0	3525.0	3533.0	3500.0	3350.0	3517.0
3567.0	3600.0	3483.0	3500.0	3550.0	3550.0	3550.0	2450.0	2800.0	3167.0
3325.0	3300.0	3317.0	3400.0	3400.0	3450.0	3433.0	3400.0	3275.0	3317.0
3550.0	3550.0	3550.0	3600.0	3617.0	3500.0	3400.0	3450.0	3450.0	3350.0
3400.0	3450.0	3550.0	3550.0	3517.0	3517.0	3500.0	3525.0	3500.0	3500.0
3350.0	3250.0	3283.0	3125.0	3150.0	3150.0	3100.0	3100.0	3217.0	3250.0
3150.0	3100.0	3000.0	3170.0	3000.0	2750.0	3200.0	3250.0	3250.0	3217.0
3200.0	3225.0	3250.0	3083.0	3200.0	3150.0	3150.0	3150.0	3100.0	3100.0
3075.0	3083.0	3100.0	3100.0	3100.0	3083.0	3100.0	3150.0	3183.0	3150.0
3283.0	3300.0	3133.0	3050.0	3100.0	3083.0	3100.0	3117.0	3150.0	3150.0
3000.0	3175.0	3050.0	3083.0	3217.0	3283.0	3150.0	3200.0	3200.0	3200.
3200.0	3150.0	3100.0	3125.0	3167.0	3200.0	2875.0	2933.0	3200.0	3175.0
3183.0	3200.0	3150.0	3125.0	3150.0	3150.0	3150.0	3117.0	3050.0	3150.0
3200.0	3250.0	3250.0	3375.0	3200.0	3333.0	3517.0	3550.0	3550.0	3517.0
3433.0	3150.0	3117.0	3250.0	3425.0	3525.0	3133.0	3450.0	3400.0	2550.0
3075.0	3533.0	3583.0	3600.0	3600.0	3550.0	3517.0	3533.0	3475.0	3550.0
3525.0	3550.0	3550.0	3550.0	3550.0	3500.0	3475.0	3500.0	3400.0	3500.0

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SERIES 4 .. HOT METAL TEMPERATURE .

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N.A.	1510.0	1510.0	1450.0	N.A.	1470.0	1420.0	1450.0	1400.0	N.A.
N.A.	N.A.	1450.0	1440.0	1450.0	1430.0	1450.0	1460.0	N.A.	1440.0
1420.0	1420.0	1470.0	1480.0	1460.0	N.A.	1470.0	1450.0	N.A.	1460.0
1480.0	1490.0	1430.0	1440.0	1430.0	1450.0	1450.0	1430.0	1430.0	1400.0
1460.0	1400.0	1430.0	N.A.	1470.0	1450.0	1470.0	1440.0	1460.0	1460.0
1430.0	1430.0	N.A.	1450.0	1430.0	N.A.	N.A.	1480.0	1420.0	1380.0
1460.0	1440.0	1400.0	1450.0	1360.0	1480.0	1460.0	1480.0	1430.0	1430.0
N.A.	N.A.	1480.0	1480.0	1470.0	N.A.	1450.0	1450.0	1460.0	N.A.
1470.0	N.A.	1470.0	N.A.	1450.0	1460.0	1440.0	N.A.	1450.0	1460.0
1450.0	1520.0	1480.0	N.A.	1460.0	1470.0	1470.0	1470.0	1460.0	1470.0
1470.0	1510.0	1450.0	1440.0	1480.0	1450.0	N.A.	N.A.	1450.0	1450.0
N.A.	1450.0	1500.0	1440.0	1470.0	1450.0	1470.0	1450.0	1490.0	1490.0
1440.0	1460.0	N.A.	N.A.	1470.0	1470.0	N.A.	1480.0	1470.0	1470.0
1500.0	N.A.	1500.0	N.A.	N.A.	1450.0	1460.0	N.A.	1460.0	1470.0
1450.0	N.A.	1470.0	N.A.	N.A.	1480.0	1490.0	N.A.	1500.0	1500.0
1480.0	1480.0	N.A.	N.A.	1490.0	1470.0	1450.0	1480.0	1450.0	1500.0
1460.0	N.A.	N.A.	N.A.	1450.0	1450.0	1450.0	1470.0	1450.0	1460.0
1470.0	1500.0	1450.0	1480.0	1450.0	1430.0	1470.0	1460.0	1490.0	1480.0
1480.0	1460.0	1480.0	1480.0	1480.0	1470.0	1380.0	1450.0	1480.0	1460.0

1480.0	1480.0	1500.0	1460.0	1350.0	1470.0	1420.0	1420.0	1480.0	1460.0
1470.0	1480.0	1480.0	1450.0	N.A.	1450.0	1440.0	1450.0	1465.0	N.A.
1480.0	N.A.	1460.0	1490.0	N.A.	1500.0	1500.0	1480.0	1480.0	1440.0
1460.0	14[illegible].0	N.A.	1450.0	1450.0	1450.0	1400.0	1450.0	1350.0	1380.0
1350.0	14[illegible].0	1400.0	1430.0	N.A.	1460.0	1460.0	1460.0	N.A.	1400.0
1460.0	N.A.	1450.0	N.A.	N.A.	1460.0	1460.0	1450.0	1450.0	1460.0
1450.0	1450.0	N.A.	1450.0	1460.0	N.A.	1460.0	1460.0	1450.0	1460.0
1470.0	1480.0	1480.0	1460.0	1470.0	N.A.	1450.0	N.A.	1450.0	1470.0
1450.0	1460.0	1480.0	N.A.	1470.0	N.A.	1450.0	N.A.	1400.0	N.A.
1360.0	1470.0	1480.0	N.A.	1475.0	1470.0	N.A.	1400.0	1470.0	N.A.
1460.0	N.A.	1450.0	N.A.	1450.0	N.A.	1480.0	N.A.	N.A.	N.A.
1480.0	1490.0	1490.0	1510.0	1460.0	1465.0	N.A.	N.A.	N.A.	1410.0
1460.0	1475.0	1440.0	140.0	1400.0	1400.0	1440.0	N.A.	N.A.	N.A.
N.A.	1450.0	1450.0	1400.0	1450.0	1430.0	N.A	1420.0	1450.0	N.A.
1460.0	N.A.	1450.0	1450.0	N.A.	N.A.	1460.0	1450.0	NOT AVALIABLE	
N.A.	N.A.	1460.0	N.A.	1450.0	N.A.	1480.0	N.A.	N.A.	1460.0

NOTE... N.A. REPRESENTS THAT DATA IS NOT AAVALIABLE.

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SERIES 5 .. SILICON CONTENT OF HOT METAL .

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1.26	1.58	1.80	1.49	1.66	1.67	1.58	1.12	1.30	1.30
1.49	1.31	1.62	1.77	1.27	1.67	1.31	1.30	1.32	1.49
1.49	1.80	1.67	1.67	1.86	1.77	1.95	1.77	1.58	1.58
1.67	1.52	1.26	1.23	1.58	1.26	1.31	1.31	1.45	1.31
1.59	1.40	1.31	1.58	1.86	1.70	1.49	1.75	1.77	1.53
1.49	1.53	1.40	1.40	1.82	1.43	1.17	1.42	1.67	1.31
1.3[illegible]	1.40	1.40	1.31	1.86	1.36	1.44	1.40	1.21	1.38
1.5[illegible]	1.35	1.40	1.10	1.40	2.00	1.31	1.31	1.63	1.46
1.2[illegible]	1.56	1.12	1.31	1.40	1.31	1.42	2.14	1.53	1.58
1.26	1.49	1.36	1.26	1.58	1.12	1.12	0.92	0.98	1.20
1.16	1.23	1.03	0.98	1.21	1.21	1.03	1.17	1.20	1.15
1.03	1.31	1.40	1.21	1.40	1.03	1.45	1.36	1.48	1.40
1.40	1.38	1.21	1.36	1.30	1.26	1.16	1.21	1.21	1.32
1.17	1.12	1.31	1.90	1.87	1.58	1.46	1.43	1.18	1.03
1.67	1.54	1.31	1.54	1.40	1.12	1.31	1.40	1.35	1.58
1.58	1.45	1.52	1.40	1.21	1.25	1.26	1.31	1.21	1.31
1.12	0.93	0.94	1.30	1.58	1.77	1.40	1.46	1.22	1.17
1.30	1.49	1.35	1.03	1.12	1.40	1.31	1.31	1.45	1.67
1.58	1.12	1.67	1.21	1.34	2.14	1.40	1.67	1.58	1.49
1.49	1.12	1.05	1.10	0.93	1.40	1.35	1.12	1.14	1.49
1.31	1.04	1.12	1.15	1.31	1.21	1.36	1.21	1.45	1.60
1.72	1.67	1.58	1.30	1.31	1.49	1.58	1.77	1.49	1.31
1.54	3.00	1.98	1.77	1.45	1.49	1.36	1.12	1.12	1.19
1.4[illegible]	1.31	1.14	1.30	1.49	1.12	1.16	1.42	1.26	1.31
1.33	1.40	1.40	1.48	1.77	1.49	1.40	1.67	1.40	1.68
1.12	0.93	0.90	1.03	1.12	1.40	1.77	1.32	1.12	0.90
1.12	1.16	1.31	1.31	1.40	1.54	1.40	1.40	1.31	1.07
1.21	1.30	1.58	1.58	1.54	1.49	1.46	1.45	1.28	1.25
1.21	1.20	1.40	1.49	1.52	1.58	1.58	1.54	1.58	1.49

1.71	1.40	1.70	1.36	1.56	1.67	1.65	1.63	1.28	1.49
1.38	1.40	1.40	1.54	1.26	1.31	1.49	1.46	1.86	1.49
1.60	1.77	1.67	1.77	1.44	1.58	1.49	1.49	1.50	1.55
1.56	1.40	1.49	1.40	1.58	1.58	1.49	1.12	1.26	1.49
1.17	1.21	1.63	1.49	1.25	1.49	1.58	1.40	1.48	1.49
1.57	1.67	1.77	1.40	1.31	1.07	1.10	1.12	1.40 ED	1.18

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SERIES 6 .. SULPHUR CONTENT OF HOT METAL.

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0.045	0.031	0.033	0.026	0.027	0.034	0.031	0.043	0.053	0.046
0.040	0.048	0.040	0.041	0.035	0.032	0.036	0.036	0.039	0.053
0.048	0.042	0.041	0.046	0.033	0.031	0.036	0.042	0.045	0.037
0.041	0.048	0.050	0.049	0.042	0.039	0.040	0.039	0.035	0.038
0.025	0.035	0.030	0.031	0.028	0.029	0.040	0.031	0.039	0.045
0.046	0.043	0.042	0.044	0.031	0.049	0.055	0.041	0.039	0.041
0.045	0.035	0.033	0.040	0.032	0.033	0.034	0.032	0.026	0.025
0.034	0.042	0.052	0.042	0.040	0.034	0.038	0.037	0.035	0.030
0.041	0.041	0.055	0.036	0.032	0.032	0.040	0.023	0.031	0.035
0.029	0.032	0.037	0.042	0.042	0.043	0.050	0.049	0.052	0.045
0.037	0.034	0.039	0.039	0.032	0.060	0.057	0.032	0.047	0.064
0.044	0.040	0.033	0.060	0.039	0.077	0.041	0.043	0.037	0.037
0.046	0.050	0.042	0.030	0.050	0.031	0.036	0.037	0.047	0.037
0.035	0.040	0.039	0.031	0.039	0.050	0.043	0.040	0.045	0.038
0.021	0.028	0.031	0.033	0.030	0.030	0.033	0.034	0.023	0.035
0.021	0.026	0.028	0.034	0.035	0.027	0.024	0.023	0.035	0.021
0.030	0.039	0.045	0.034	0.028	0.032	0.030	0.032	0.045	0.058
0.044	0.040	0.039	0.042	0.042	0.050	0.054	0.043	0.043	0.036
0.033	0.048	0.045	0.048	0.043	0.031	0.050	0.039	0.033	0.039
0.035	0.042	0.045	0.058	0.050	0.044	0.055	0.057	0.039	0.041
0.026	0.042	0.040	0.040	0.032	0.038	0.037	0.040	0.031	0.026
0.030	0.032	0.038	0.042	0.037	0.028	0.031	0.024	0.032	0.049
0.042	0.022	0.032	0.035	0.028	0.023	0.032	0.032	0.056	0.036
0.038	0.046	0.037	0.035	0.026	0.037	0.040	0.032	0.052	0.038
0.048	0.035	0.044	0.038	0.037	0.037	0.031	0.032	0.035	0.045
0.042	0.058	0.059	0.048	0.043	0.037	0.031	0.042	0.047	0.065
0.045	0.058	0.029	0.042	0.028	0.039	0.047	0.042	0.033	0.052
0.049	0.031	0.039	0.035	0.043	0.035	0.045	0.042	0.042	0.046
0.054	0.055	0.047	0.039	0.045	0.042	0.043	0.041	0.043	0.045
0.054	0.055	0.046	0.050	0.043	0.027	0.035	0.035	0.043	0.040
0.046	0.038	0.038	0.039	0.037	0.036	0.036	0.037	0.044	0.027
0.032	0.028	0.035	0.044	0.048	0.043	0.048	0.041	0.037	0.031
0.042	0.048	0.050	0.041	0.040	0.049	0.039	0.054	0.045	0.056
0.048	0.045	0.037	0.040	0.050	0.038	0.031	0.046	0.033	0.035
0.032	0.036	0.040	0.039	0.034	0.043	0.035	0.045	0.041ED	0.043

-

```
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
              A P P E N D I X   D
              -----------------------------
    ******  COMPUTER PROGRAMS ******


* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
   PROGRAM NO. 1   TO CALCULATE AUTOCORRELATION AND PARTIAL AUTOCORRELATION
                         FUNCTION OF THE SERIES ..
** * * * * * * * * * * * * * *  * * * * * * * * * *  * * * * * * * * * * * * * *
** **   NOTATIONS  .*-*-*-*-*
      ACVF(I) ...   AUTOCOVARIANCE FUNCTUON.
      ACF(I) ...   AUTOCORRELATION FUNCTION.
      PACF(I,I) ...   PARTIAL AUTOCORRELATION FUNCTION.
      MT ...   NUMBER OF PROBLEMS.
      N ...   NUMBER OF DATA POINTS.
      SMEAN ...   MEAN OF THE SERIES.
      SV ...   VARIANCEOOF THE SERIES.
      SD ...   STANDARD DEVIATION OF THE SERIES.
      WBAR(I) ...   STANDARDIZED SERIES.
      NDIF ...   DEGREE OF DIFFERENCING.
** * * * * * * * * * * * * * *  * * * * * * * * * *  * * * * * * * * * * * * * *
      DIMENSION WW(1050)
      DOUBLE PRECISION WBAR(1050)
      DOUBLE PRECISION W(1050),ACVF(100),ACF(100),PACF(50,50)
      DOUBLE PRECISION SBAR,SMBAR,SSBAR,SVBAR,SDBAR
      DOUBLE PRECISION SM,SMEAN,SS,SV,SD,SNUM,SDEN
      IACF=50
      IPACF=50
      READ 101,MT
      DO 1000 NTS=1,MT
      READ101,N
      READ 102,(WW(I),I=1,N)
      DO 1 I=1,N
    1 W(I)=DBLE(WW(I))
      PRINT103
      IF(NTS.EQ.1) PRINT 121
      IF(NTS.EQ.2) PRINT 122
      IF(NTS.EQ.3) PRINT 123
      IF(NTS.EQ.4) PRINT 124
       IF(NTS.EQ.5) PRINT 125
      IF(NTS.EQ.6) PRINT 126
C      ************************************************************************
C    CALCULATE MEAN AND STANDARD DEVIATION OF THE ORIGINAL SERIES.
C      ************************************************************************
      SM=0.0D0
      DO 5 I=1,N
    5 SM=SM+W(I)
      SMEAN=SM/DBLE(FLOAT(N))
      SS=0.0D0
```

```
      DO 6 I=1,N
    6 SS=SS+(W(I)-SMEAN)**2
      SV=SS/DBLE(FLOAT(N))
      SD=DSQRT(SV)
      PRINT105,N
      PRINT 106
      PRINT107,SMEAN,SV,SD

C     TRANSFORM THE SERIES BY SUBTRACTING MEAN AND DIVIDING BY STANDARD DEVIATION.

      DO 3 I=1,N
    3 WBAR(I)=(W(I)-SMEAN)/SD
      DO12 ID=1,3
      NDIF=ID-1
      IF(NDIF.EQ.0)GO TO 4
      N=N-1
      DO 2 I=1,N
    2 WBAR(I)=WBAR(I+1)-WBAR(I)
    4 CONTINUE

C     CALCULATE MEAN AND STANDARD DEVIATION OF TRANSFORMED AND DIFFERENCED, IF
C     (NDIF .EQ.2 AND 3), SERIES.

      SBAR=0.0D0
      DO 30 I=1,N
   30 SBAR=SBAR+WBAR(I)
      SMBAR=SBAR/DBLE(FLOAT(N))
      SSBAR=0.0D0
      DO 31 I=1,N
   31 SSBAR=SSBAR+(WBAR(I)-SMBAR)**2
      SVBAR=SSBAR/DBLE(FLOAT(N))
      SDBAR=DSQRT(SVBAR)
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     CALCULATE AUTO CORRELATION FUNCTION.
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DO 8 I=1,IACF
      ACVF(I)=0.0D0
      NM=N-I
      DO 7 J=1,NM
      JJ=J+I
    7 ACVF(I)=ACVF(I)+(WBAR(J)-SMBAR)*(WBAR(JJ)-SMBAR)
      ACVF(I)=ACVF(I)/DBLE(FLOAT(N))
      ACF(I)=ACVF(I)/SVBAR
    8 CONTINUE
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     CALCULATE PARTIAL AUTO CORRELATION FUNCTION
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      PACF(1,1)=ACF(1)
      DO11 L=2,IPACF
      LL=L-1
```

```
      SNUM=0.0D0
      SDEN=0.0D0
      DO 9 J=1,LL
      LJ=L-J
      LLL=L-1
      SNUM=SNUM+PACF(LLL,J)*ACF(LJ)
    9 SDEN=SDEN+PACF(LLL,J)*ACF(J)
      PACF(L,L)=(ACF(L)-SNUM)/(1.0D0-SDEN)
      DO 10 J=1,LL
      LJJ=L-J
      LN=L-1
   10 PACF(L,J)=PACF(LN,J)-PACF(L,L)*PACF(LN,LJJ)
   11 CONTINUE
      PRINT104,NDIF
      PRINT 158
      PRINT 159,SMBAR,SVBAR,SDBAR

C     PRINT THE AUTO COVARIANCE FUNCTION)

      PRINT 150
      DO 20 J=1,IACF,50
      IE=J+49
   20 PRINT 151,(ACVF(I),I=J,IE)

C     PRINT THE AUTO CORRELATION FUNCTION.

      PRINT 152
      DO 21 J=1,IACF,50
      IE=J+49
   21 PRINT 151,(ACF(I),I=J,IE)

C     PRINT THE PARTIAL AUTO CORRELATION FUNCTION.

      PRINT 154
      DO 14 J=1,IPACF,50
      IE=J+49
   14 PRINT 151,(PACF(I,I),I=J,IE)
   13 CONTINUE
 1000 CONTINUE
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
           INPUT - OUTPUT FORMAT STATEMENTS.
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
XCEEDS SPECIFIED LIMIT JOB TERMINATED
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

   PROGRAM NO. 2   TO CALCULATE POWER SPECTRA OF THE SERIES ..

C    SPECTRAL ANALYSIS OF TIME SERIES ...
C    THIS PROGRAM USES HAMMING-TUKEY METHOD OF SMOOTHING.
C    THIS PROGRAM ALSO CALCULATES CROSS-CORRELATION AND AUTO-CORRELATION FUNCTIO
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*-*-*  NOTATIONS.
     WP(I)   AUTOCOVARIANCE FUNCTION.
       RP(I)   AUTOCORRELATION FUNCTION.
       FLP(I)   RAW SPECTRA.
      UP(I)   SMOOTH SPECTRA.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
       DIMENSION X(6,350),TP(350),SP(350),FP(350),GP(350),Z(6,350),
      1ZZ(6,350),CP(40),WP(40),RP(40),UP(40),FLP(40)
       READ 101,NN,NVAR
       DO 809 LM=1,NVAR
       IF (LM.LE.2) READ 102,( Z(LM,I),I=1,NN)
       IF (LM.EQ.3) GO TO 1111
       IF (LM.GE.4) READ 102,( Z(LM,I),I=1,NN)
       GO TO 809
 1111  READ 102,( Z(LM,I),I=1,40)
       READ 103,( Z(LM,I),I=41,43)
       READ 102,( Z(LM,I),I=44,NN)
  809  CONTINUE
       READ 101,NPROB
       DO 6000 LMM=1,NPROB
       DO 300 IX=1,NVAR
       DO 300 IY=1,NVAR
       SUMX=0.0
       SUMY=0.0
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C    CALCULATE MEAN AND STANDARD DEVIATION OF THE SERIES.
       DO 301 I=1,NSAMP
       SUMX=SUMX+Z(IX,I)
       SUMY=SUMY+Z(IY,I)
  301  CONTINUE
       XMEAN=SUMX/FLOAT(NSAMP)
       YMEAN=SUMY/FLOAT(NSAMP)
       SVX=0.0
       SVY=0.0
       DO 302 I=1,NSAMP
       SVX=SVX+(Z(IX,I)-XMEAN)**2
       SVY=SVY+(Z(IY,I)-YMEAN)**2
  302  CONTINUE
       SVX=SVX/FLOAT(NSAMP)
       SVY=SVY/FLOAT(NSAMP)
       SDX=SQRT(SVX)
```

```
      SDY=SQRT(SVY)
      PRINT 304,XMEAN,YMEAN,SVX,SVY,SDX,SDY
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C     STANDARDIZE THE SERIES.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
      DO 303 I=1,NSAMP
      X(IX,I)=(Z(IX,I)-XMEAN)/SDX
      X(IY,I)=(Z(IY,I)-YMEAN)/SDY
  303 CONTINUE
  204 CONTINUE
      MLAG2=MLAG-1
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  CALCULATE THE AUTOCOVARIANCE AND AUTOCORRELATION FUNCTIONS.
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      TP(1)=0.0
      GP(1)=0.0
      FP(1)=0.0
      SP(1)=0.0
      DO 16 I=1,NSAMP
      SP(1)=SP(1)+X(IX,I)**2
      TP(1)=TP(1)+X(IX,I)
      GP(1)=GP(1)+X(IY,I)**2
      FP(1)=FP(1)+X(IY,I)
   16 CONTINUE
      M1=MLAG+1
      DO 20 I=2,M1
      J=I-1
      K=NSAMP-I+2
      TP(I)=TP(J)-X(IX,J)
      SP(I)=SP(I-1)-X(IX,J)**2
      FP(I)=FP(J)-X(IY,K)
      GP(I)=GP(I-1)-X(IY,K)**2
   20 CONTINUE
      MLAG1=MLAG+1
      DO 30 I=1,MLAG1
      NMINP=NSAMP-I+1
      CP(I)=0.0
      DO 26 J=1,NMINP
      K2=J+I-1
   26 CP(I)=CP(I)+(X(IX,K2)*X(IY,J))
      WP(I)=CP(I)/FLOAT(NMINP)
      RNUM=FLOAT(NMINP)*CP(I)-FP(I)*TP(I)
      RDEN1=SQRT((FLOAT(NMINP)*GP(I))-FP(I)**2)
      RDEN2=SQRT((FLOAT(NMINP)*SP(I))-TP(I)**2)
   30 RP(I,IX,IY)=RNUM/(RDEN1*RDEN2)

C     CALCULATE THE RAW SPECTRA.

   33 DO 39 I=1,MLAG1
      FLP(I)=0.0
```

```
      DO 36 J=2,MLAG
   36 FLP(I)=FLP(I)+2.*WP(J)*COS(3.1415927*FLOAT((I-1)*(J-1))/FLOAT(MLAG
     1))
   39 FLP(I)=FLP(I)+WP(1)+WP(MLAG1)*COS(3.1415927*FLOAT(I-1))
C     SMOOTHEN THE SPECTRA.
      UP(1)=0.46*FLP(2)+0.54*FLP(1)
      UP(MLAG1)=0.46*FLP(MLAG)+0.54*FLP(MLAG1)
      DO 43 I=2,MLAG
   43 UP(I)=0.23*FLP(I-1)+0.54*FLP(I)+0.23*FLP(I+1)
      PRINT 98,IX,IY
      DO 75 I=1,MLAG1
      IP=I-1
   75 PRINT 100,IP,WP(I),FLP(I),UP(I),RP(I)
  300 CONTINUE
 6000 CONTINUE
  100 FORMAT(1X,I5,5E19.8,F16.8)
  101 FORMAT(10I5)
  102 FORMAT(5F16.7)
  103 FORMAT(5F16.7)
  304 FORMAT(1X,*XMEAN=*,E20.8,5X,*YMEAN=*,E20.8,5X,*SVX=*,E20.8/,1X,*SV
     2Y=*,E20.8,5X,*SDX=*,E20.8,5X,*SDY=*,E20.8)
   98 FORMAT(2X,* SPECTRAL ANALYSIS OF*,I4,*VS*,I4,//*    P
     1COVAR         RAW SPECTRA    SMOOTH SPECTRA *//)
      STOP
      END
```
* *

```
***  OBJECT PROGRAM IS BEING ENTERED INTO STORAGE AT  05 HRS.  50 MTS.  32 SEC
```

```
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
   PROGRAM NO. 3   TO CALCULATE THE INITIAL ESTIMATES OF THE PARAMETERS ..
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
***   THIS PROGRAM CALCULATES INITIAL ESTIMATES FOR UNIVARIATE STOCHASTIC
***   MODEL OF A TIME SERIES.  THE ALGORITHM IS GIVEN IN REFERENCE NO. 65.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
***   NOTATIONS.......
***   PHI(I) ... INITIAL ESTIMATES OF AR PARAMETERS.
***   THETA(I) ...  INITIAL ESTIMATES OF MA PARAMETERS.
***   ITMAX ...   MAXIMUM NO. OF ITERATIONS.
***   NPROB ... NO. OF PROBLEMS.
***   SM ...   MEAN OF THE DATA SERIES.
***   C(I) ...   AUTOCOVARIANCES OF THE SERIES.
***   NP ...   ORDER OF AUTOREGRESSSION.
***   NQ ...   ORDER OF MOVING AVERAGE.
***   NDIF ...   DEGREE OF DIFFERENCING.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C     UNIVARIATE STOCHASTIC MODEL PRELIMINARY ESTIMATION (USPE)
      DIMENSION PHI(6),THETA(6),C(10),TAU(6),F(6),X(6),H(6),CH(10)
      DIMENSION A(6,6),T(6,6),T1(6,6),T2(6,6)
      ETA=0.001
      ITMAX=100
      READ 1,NPROB
      DO 6000 L=1,NPROB
      READ 2,NP,NDIF,NQ
       READ 3,SM
      NPQ=NP+NQ+1
      READ 4,(C(I),I=1,NPQ)
C   PRINT THE INPUT
      PRINT 204 ,NP,NDIF,NQ
      PRINT 205,SM
      PRINT 206,(C(I),I=1,NPQ)
      IF(NQ.GT.0) PRINT 401
      IF(NQ.GT.0) PRINT 149
      NQQ=NQ+1
      NPP=NP+1
      IF(NP.EQ.0) GO TO 300

C   CALCULATE THE INITIAL ESTIMATES PHI OF AUTOREGRESSIVE PARAMETERS.

      DO 6 I=1,NP
      IQ=NQ+I+1
      X(I)=C(IQ)
      DO 6 J=1,NP
      IJQ=IABS(NQ+I-J)+1
    6 A(I,J)=C(IJQ)

C   SOLVE THE SET OF NP LINEAR EQUATIONS.

C   SUBROUTINE MATINV CALCULATES THE ROOTS OF THE SIMULTANIOUS EQUATIONS.
```

```
      CALL MATINV (A,NP,X,1,DETER)
      DO 15 I=1,NP
   15 PHI(I+1)=X(I)
      PHI(1)=-1.0
      DO 33 J=1,NQQ
      CH(J)=0.0
      DO 24 I=1,NPP
      DO 24 K=1,NPP
      IJK=I+J-K
      IF(IJK.LE.0) IJK=IABS(IJK)+2
   24 CH(J)=CH(J)+PHI(I)*PHI(K)*C(IJK)
   33 CONTINUE
      GO TO 100
  300 DO 42 J=1,NQQ

C     CALCULATE INITIAL ESTIMATES THETA OF MOVING AVERAGE PARAMETERS.
C     THIS UTILIZES NEWTON-RAPHSON ALGORITHM.

   42 CH(J)=C(J)
  100 IF(NQ.EQ.0) GO TO 2400
C     THE ITERATIONS BEGIN HERE.
      IT=1
      IF(CH(1).LT.0.) GO TO 111
      GO TO 112
  111 PRINT 222,CH(1)
      GO TO 6000
  112 TAU(1)=SQRT(CH(1))
      DO 51 I=1,NQ
   51 TAU(I+1)=0.0
  240 DO 60 J=1,NQQ
      F(J)=0.0
      JQ=NQQ+1-J
      DO 69 I=1,JQ
      IJ=I+J-1
   69 F(J)=F(J)+TAU(I)*TAU(IJ)
      F(J)=F(J)-CH(J)
   60 CONTINUE

C     FORM THE MATRIX T OF TAU(I).

      DO 78 I=1,NQQ
      DO 78 J=1,NQQ
      T1(I,J)=0.0
   78 T2(I,J)=0.0
      NNQ=NQQ
      II=0
      DO 87 I=1,NQQ
      DO 96 J=1,NNQ
      II=II+1
   96 T1(I,J)=TAU(II)
```

```
      NNQ=NNQ-1
      II=I
   87 CONTINUE
      NNQ=NQQ
      II=0
      DO 105 J=1,NQQ
      DO 114 I=1,NNQ
      II=II+1
  114 T2(J,II)=TAU(I)
      NNQ=NNQ-1
      II=J
  105 CONTINUE
      DO 123 I=1,NQQ
      DO 123 J=1,NQQ
  123 T(I,J)=T1(I,J)+T2(I,J)

C     SOLVE THE SET OF (NQ+1) LINEAR EQUATIONS.

      CALL MATINV (T,NQQ,F,1,DETER)
      DO 132 I=1,NQQ
  132 H(I)=F(I)

C     CALCULATE MOVING AVERAGE PARAMETERSA

      DO 141 J=1,NQ
  141 THETA(J+1)=-TAU(J+1)/TAU(1)
      PRINT 150,IT,(THETA(J),J=2,NQQ)
      DO 159 I=1,NQQ
  159 TAU(I)=TAU(I)-H(I)
      IT=IT+1
      IF(IT.LT.ITMAX) GO TO 160
      PRINT 600
      GO TO 2400
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C     CONVERGENCE TESTING.
  160 DO 169 I=1,NQQ
  169 IF(ABS(F(I)).GT.ETA) GO TO 240
C     IF ALL F(I).LT. ETA ASSUME CONVERGENCE.
C     CALCULATE INITIAL ESTIMATE OF OVERALL CONSTANT THETA(0).

 2400 IF(NP.EQ.0) GO TO 330
      PHIS=0.0
      DO 177 I=1,NP
  177 PHIS=PHIS+PHI(I+1)
      THETO=SM*(1.0-PHIS)
      GO TO 200
  330 THETO=SM

C     CALCULATE INITIAL ESTIMATE OF WHITE NOISE VARIANCE.
```

```
  200 IF(NQ.GT.0) GO TO 186
      PHIC=0.0
      DO 195 I=1,NP
  195 PHIC=PHIC+PHI(I+1)*C(I+1)
      VAR=C(1)-PHIC
      GO TO 310
  186 VAR=TAU(1)**2
  310 IF(NP.EQ.0) GO TO 210
      PRINT 208,(PHI(I+1),I=1,NP)
  210 CONTINUE
      PRINT 211,THETO
      PRINT 212,VAR
 6000 CONTINUE
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
                FORMAT FOR INPUT - OUTPUT.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
    1 FORMAT(10I5)
    2 FORMAT(3I2)
    3 FORMAT(5F12.7)
    4 FORMAT(6F12.7)
  149 FORMAT(//5X,*ITERATION*,12X,*THETA(I)*//)
  150 FORMAT(5X,I5,5E20.7)
  204 FORMAT(//1X,*ORDER OF THE PROCESS IS ( *,3I2,* )*)
  205 FORMAT(//1X,*THE MEAN OF THE SERIES IS=*,F12.7)
  206 FORMAT(//1X,*AUTO COVARIANCES OF THE SERIES ARE*,5F15.7)
     3EACH ITERATION ARE AS FOLLOWS*//)
  208 FORMAT(//1X,*INITIAL ESTIMATES OF AUTOREGRESSIVE PARAMETERS ARE*,3
     1E20.7)
  211 FORMAT(//1X,*INITIAL ESTIMATE OF OVERALL CONSTANT TERM IS *,E15.7)
  212 FORMAT(//1X,*INITIAL ESTIMATE OF WHITE NOISE VARIANCE IS *,E15.8)
  222 FORMAT(1X,*TAU0 IS NEGATIVE TAKE NEXT PROBLEM*,E17.8)
  401 FORMAT(//1X,*INITIAL ESTIMATES OF MOVING AVERAGE PARAMETERS AT
  600 FORMAT(//5X,*NO. OF ITERATIONS EXCEEDS THE LIMIT HENCE PROGRAM IS
     6TERMINATED*)

      STOP
      END
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
```

```
   PROGRAM NO.  4    FINAL ESTIMATICN OF THE PARAMETERS.
C    ITERATIVE METHOD FOR PARAMETER ESTIMATION
C    UTILIZES MARQUARDT ALGORITHM FOR NON-LINEAR LEAST SQUARES


*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C   * * *    MAIN    PRCGRAM    * * *
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
*-*-*-    N O T A T I O N S *-*-*-*-*-
**  NQ  ORDER OF MOVING AVERAGE PROCESS.
   NP  ORDER CA AURCREGRESSION .
   N    LENGTH OF THE SERIES.
   NDIF   DEGREE OF DIFFERENCING.
   ORIGIN   INTEGER VARIABLE.
   KK    INTEGER VARIABLE
             KK= 1   PROGRAM IS USED BY MAIN PROGRAM TO CALCULATE AA(T) AND AA(T)
                 SQUARE  PHI AND THETA ENTERED AS ARRAY BETA.
                KK = 2  PROGRAM IS USED BY MAIN PROGRAM TO CALCULATE AA(T) FOR
                 BETA PETTURBED BY STALL QUANTITY DEL.
   THETA(I)   ARRAY OF MOVING AVERAGE PARAMETERS.
   PHI(I)    ARRAY OF AUTOREGRESSIVE PARAMETERS.
   AA(T)    UNIVARIATE RESIDUAL SERIES.
   USS    DOUBLE PRECISION VARIABLE , UNCOBDINITIONAL SUM OF SQUARES.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
      DOUBLE PRECISION USS,USSO
      DOUBLE PRECISION  UXX
      INTEGER ORIGIN
      DIMENSION ACF(60)
      DIMENSION WW(375),[illegible](375),W(375),Z(375),PHI(4),THETA(4),BETA(4)
      DIMENSION WWW(375)
      DIMENSION WC(375)
      DIMENSION XX(3,375),AA(375),G(4),BB(375),S(4),BETAD(4),B(4,1)
      DIMENSION AAA(4,4),AAAS(4,4),D(4),GS(4),R(4,4),V(4,4),H(4),HS(4)
      COMMON WW
      READ 101,NPROB
      DO 1500 LM=1,NPROB
      READ 101,N
      NN=N
      READ 102,(Z(I),I=1,N)

C     CALCULATE MEAN AND STANDARD DEVIATION

      SUM=0.0
      DO 100 I=1,N
  100 SUM=SUM+Z(I)
      SMEAN=SUM/FLOAT(N)
       SSS=0.0
      DO 1[illegible] I=1,N
```

```
104 SSS=SSS+(Z(I)-SMEAN)**2
    SVA=SSS/FLOAT(N)
    SD=SQRT(SVA)
    DO 105 I=1,N
105 WWW(I)=(Z(I)-SMEAN)/SD
    READ 3,NCRD
    DO 20  LMMN=1,NCRD
    READ 100,NP,NDIF,NQ
    IF(NP.GT.0) READ 4,(PHI(I),I=1,NP)
  4 FORMAT(8F10.7)
    IF(NQ.GT.0) READ 5,(THETA(I),I=1,NQ)
  5 FORMAT(8F10.7)
    IF(NDIF-1) 111,106,108
111 DO 112 I=1,MN
112 WO(I)=WWW(I)
    GO TO 110
106 DO 113 I=1,MN
113 WO(I)=WWW(I)
    N=MN-1
    DO 107 I=1,N
107 WO(I)=WO(I+1)-WO(I)
    GO TO 110
108 DO 114 I=1,MN
114 WO(I)=WWW(I)
    N=MN-2
    DO 109 I=1,N
109 WO(I)=WO(I+2)-2.0*WO(I+1)+WO(I)
110 CONTINUE
    SV=0.
    SM=0.0
    DO 710  I=1,N
710 SM=SM+WO(I)
    SM=SM/FLOAT(N)
    DO 711 I=1,N
711 SV=SV+(WO(I)-SM)**2
    SV=SV/FLOAT(N)
    SD=SQRT(SV)
    PRINT 7,SM,SV
    IF(NDIF.EQ.0) GO TO 3008
    DO 3004 I=1,N
3004 W(I)=(WO(I)-SM)/SD
    SMBAR=0.0
    SVBAR=0.0
    DO 3005 I=1,N
3005 SMBAR=SMBAR+W(I)
    SMBAR=SMBAR/FLOAT(N)
    DO 3006 I=1,N
3006 SVBAR=SVBAR+(W(I)-SMBAR)**2
    SVBAR=SVBAR/FLOAT(N)
    PRINT 3007,SMBAR,SVBAR
```

```
[illegible] FORMAT(1X,*MEAN OF TRANSFORMED SERIES=*,E20.7/1X,*VARIANCE=*,E20.7
     1)
      GO TO [illegible]10
[illegible] DO [illegible] I=1,N
[illegible] W(I)=W[illegible](I)
[illegible] CONTINUE
      DO [illegible] I=1,[illegible]75
[illegible] WW(I)=[illegible].[illegible]
      [illegible]=
      PI=[illegible].[illegible]1
      DEL=[illegible].[illegible]1
      ETA=[illegible].[illegible]1
      F2=4.[illegible]
      PIMAX=[illegible].[illegible]
      KOUNT=
      [illegible]=1

C     SET UP BETA

      IF(NP.EQ.0) GO TO 6[illegible]1
      DO [illegible] I=1,NP
  6[illegible]  BETA(I)=PHI(I)
  6[illegible]1 IF(NQ.EQ.0) GO TO 6[illegible]3
      DO 6[illegible]4 I=1,NQ
      NPI=NP+I
  6[illegible]4 BETA(NPI)=THETA(I)
  6[illegible] NB=NP+NQ+MEAN
      IF(MEAN.EQ.1) BETA(NB)=SM

C     GET AA SERIES

  6[illegible] KK=1
      CALL UNCONS(NP,NQ,N,NN,NDIF,KK,ORIGIN,KOUNT,SM,PHI,THETA,XX,DEL,BE
     1TA,AA,W,MEAN,USS)
      USSO=USS

C     GET PETURBED AA SERIES (IN XX)

      KK=2
      CALL UNCONS(NP,NQ,N,NN,NDIF,KK,ORIGIN,KOUNT,SM,PHI,THETA,XX,DEL,BE
     1TA,AA,W,MEAN,USS)
      DO 2[illegible] I=1,NN
  2[illegible] BB(I)=[illegible](1,I)
      DO [illegible] J=1,NN
      DO [illegible] I=1,NB
  6[illegible]5 [illegible](I,J)=(AA(J)-XX(I,J))/DEL

C     PRINTOUT FIRST RUN THROUGH BACKFORECASTING AND FORWARD RECURSION

      IF(KOUNT.GT.4) GO TO 522
```

```
      PRINT 515
      PRINT 516,NP,NDIF,NQ
      IF(NP.GT.0) PRINT 517,(PHI(I),I=1,NP)
      IF(NQ.GT.0) PRINT 517,(THETA(I),I=1,NQ)
      IF(MEAN.EQ.1) PRINT 46,SM
      IF(MEAN.EQ.0) PRINT 47
      PRINT 518
      NEND=IL+ORIGIN
      DO 521 I=1,NEND
      ITT=I+ORIGIN
  521 CONTINUE
  522 CONTINUE
C     BEGINNING OF ITERATIONS
      DO 631 I=1,NB
      G(I)=0.0
      DO 632 J=1,NB
      AAA(I,J)=0.0
      DO 632 JJ=1,NN
  632 AAA(I,J)=AAA(I,J)+XX(I,JJ)*XX(J,JJ)
      DO 633 JJ=1,NN
  633 G(I)=G(I)+XY(I,JJ)*AA(JJ)
      D(I)=SQRT(AAA(I,I))
  631 CONTINUE
  650 CONTINUE
      IF(IT.GT.1) GO TO 685
      PRINT 686,NP,NDIF,NQ
      PRINT 619,PI,PIMAX,F2,ETA,DEL
      PRINT 687
  685 CONTINUE
      PRINT 688,IT,USS,(BETA(I),I=1,NB),PI
      IT=IT+1
      IF(IT.LT.51) GO TO 690
      PRINT 691
      GO TO 670
  690 CONTINUE

C     CONSTRUCT MODIFIED (SCALED AND CONSTRAINED) LINEARIZED EQUATIONS

      DO 651 I=1,NB
      AAAS(I,I)=1.0+PI
      GS(I)=G(I)/D(I)
      DO 651 J=1,NB
      IF(I.EQ.J) GO TO 651
      AAAS(I,J)=AAA(I,J)/(D(I)*D(J))
  651 CONTINUE

C     SOLVE FOR H

      IF(NB.EQ.1) GO TO 652
```

```
C     INVERT AAAS

      CALL MATINV(AAAS,NB,B,0,DETER)
      IF(DETER.NE.0.) GO TO 655
      PRINT 854
      STOP
  655 AAAS(1,1)=1./AAAS(1,1)
  659 CONTINUE
      DO 656 I=1,NB
      HS(I)=0.0
      DO 657 J=1,NB
  657 HS(I)=HS(I)+AAAS(I,J)*GS(J)
      H(I)=HS(I)/D(I)
  656 CONTINUE
      DO 66 I=1,NB
      BETAC(I)=BETA(I)
   66 BETA(I)=BETAC(I)+H(I)
      KK=1
      CALL UNCONS(NP,NQ,N,NN,NDIF,KK,ORIGIN,KOUNT,SM,PHI,THETA,XX,DEL,BE
     1TA,AA,W,MEAN,USS)
      IF(USS.GT.USSC) GO TO 661

C     CONVERGENCE TESTING

      DO 662 I=1,NB
      IF(ABS(H(I)).GE.ETA) GO TO 663
  662 CONTINUE

C     IF ALL H(I).LT.ETA , ASSUME CONVERGENC*

      GO TO 670
  664 BETAC(I)=BETA(I)
      PI=PI/F2
  661 DO 664 I=1,NB
      GO TO 63
  663 PI=PI*F1
      DO 69 I=1,NB
   69 BETA(I)=BETAC(I)
      DO 40 I=1,NB
      IF(ABS(H(I)).GT.ETA) GO TO 41
      CONTINUE

C     CHECK AROUND APPARENT MINIUM

      PRINT 81
      DO 42 I=1,NB
      DO 42 J=1,2
      BETA(I)=BETA(I)-ETA*((-1.0)**J)
      KK=1
      CALL UNCONS (NP,NQ,N,NN,NDIF,KK,ORIGIN,KOUNT,SM,PHI,THETA,XX,DEL,B
```

```
     1ETA,PB,W,MEAN,UXX)
      PRINT 686,IT,UXX,(BETA(K),K=1,NB)
      BETA(I)=BETA(I)+ETA*((-1.0)**J)
      IT=IT+1
  [illegible] CONTINUE
      GO TO 67[illegible]
  [illegible] CONTINUE
      IF(PI.LT.PIMAX) GO TO 655
      PRINT 666
  67[illegible] CONTINUE
C     COMPUTE RESIDUAL VARIANCE AND COVARIANCE MATRIX
      PRINT 688,IT,USS,(BETA(I),I=1,NB),PI
      PRINT 3[illegible],IT
      PRINT 692
  692 FORMAT(1H,*RESIDUAL AA(I)*)
      PRINT 1[illegible]6,(AA(I),I=1,NN)
      NS=NN-ORIGIN
      ORIGIN=NQ
      IF(NP.GT.0) ORIGIN=NP+NQ+20
      NBB=ORIGIN+1
      PUNCH [illegible]15,(AA(I),I=NBB,NN)
      NZZ=N[illegible]P-NQ
      SVAR=USS/FLOAT(NZZ)
      PRINT [illegible],SVAR
      DO 671 I=1,NB
      DO 671 J=1,NB
      V(I,J)=[illegible]
      DO 67[illegible] JJ=1,NN
  67[illegible] V(I,J)=V(I,J)+XX(I,JJ)*XX(J,JJ)
C     INVERT V(I,J)
      IF(NB.EQ.1) GO TO 672
      CALL MATINV(V,NB,B,[illegible],DETER)
      IF(DETER.NE.0.) GO TO 673
      PRINT 674
      GO TO 9
  672 V(1,1)=1.0/V(1,1)
  673 CONTINUE
      DO 675 I=1,NB
      DO 676 J=1,NB
      V(I,J)=V(I,J)*SVAR
  676 R(I,J)=V(I,J)/SQRT(V(I,I)*V(J,J))
  675 S(I)=SQRT(V(I,I))
      PRINT 696,(S(I),I=1,NB)
C     EVALUATE CONSTANT TERM
```

```
      IF(MEAN.EQ.0) GO TO 677
      THETC=SM
      IF(NP.EQ.0) GO TO 677
      THETC=0.
      DO 679 I=1,NP
  678 THETC=THETC+PHI(I)
  679 THETC=THETC+PHI(I)
      THETC=SM*(1.0-THETC)
      PRINT 10,THETC
  677 CONTINUE
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C     DIAGNOSTIC CHECK
C     COMPUTE RESIDUAL ACF
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
      CHISQ=0.0
      AM=0.
      NS=ORIGIN+1
      DO 681 I=NS,NN
  681 AM=AM+AA(I)
      AM=AM/(FLOAT(NN)-FLCAT(ORIGIN))
      KEND=20
      KE=KEND+1
      DO 682 KKK=1,KE
      CV=0.
      JS=ORIGIN+1
      JE=NN-KKK+1
      DO 680 J=JS,JE
      KKKK=KKK+J
  680 CV=CV+(AA(J)-AM)*(AA(KKKK-1)-AM)
      IF(KKK.EQ.1) GO TO 706
      CV=CV/(FLOAT(NN)-FLOAT(ORIGIN))
      ACF(KKK-1)=CV/CVC
      IF(KKK.LE.21) CHISQ=CHISQ+ACF(KKK-1)**2
      GO TO 682
  706 CV=CV/(FLCAT(N)-FLCAT(NB))
      CVC=CV
  682 CONTINUE
      CHISQ=CHISQ*(FLOAT(NN)-FLOAT(ORIGIN))
      IDOF=20-NB
      JS=ORIGIN+1
      PRINT 694,CVC
      PRINT 695
      DO 697 I=1,NB
      PRINT 693,(V(I,J),J=1,NB)
  693 FORMAT(1X,8E15.7)
  697 CONTINUE
  694 FORMAT(1X,*STANDARD ERRORS ARE*/5X,8(E15.7,1X))
      PRINT 698
      DO 699 I=1,NB
  699 PRINT 693,(R(I,J),J=1,NB)
```

```
      PRINT 7 2
      DO 7 [illegible] J=1,KEND,12
      I =J+11
      IEE=I
      IF(IEE.GT.KEND) IEE=KEND
  7 2 PRINT 7 [illegible],(I,I=J,IE),(ACF(I),I=J,IEE)
      PRINT 7 5,CHISQ,IDOF
      S =SQRT(1.0/(FLOAT(NN)-FLOAT(ORIGIN)))
      PRINT 7 9,S
      PCVR=(CVO/SV)*100.0
      PRINT 712,SV,PCVR
    9 CONTINUE
  24  CONTINUE
  15    CONTINUE
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
                 FORMAT STATEMENTS.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
  1   FORMAT(1I2)
  1 1 FORMAT(10I5)
  1 2 FORMAT(10F8.3)
 [illegible]2 FORMAT(1X,*THE NO. OF OBSEARVATIONS IN TIME SERIES=*,I8)
 [illegible]1 FORMAT(1X,21F6.2)
 30   FORMAT(1X,*THE ORIGINAL SERIES*)
    7 FORMAT(1X,*THE MEAN OF THE TRANSFORMED SERIES IS*,E20.7,5X,*THE VA
     1RIANCE IS*,E20.7)
      FORMAT(1X,10E13.5)
    2 FORMAT(1X,*THE DIFFERENCED SERIES*)
      FORMAT(1X,*USSC=*,D20.8)
      FORMAT(4E20.8)
      FORMAT(1X,*UNCONDITIONAL ESTIMATION OF AA(I) SERIES AND HENCE SUM
     1OF SQUARES*)
  5   FORMAT(1X,*ORDER OF PROCESS IS(*,3I2,*)*//1X,*PARAMETERS ARE*//)
  4 6 FORMAT(//1X,*INITIAL VALUE ASSUMED FOR MEAN=*,E20.7)
  4   FORMAT(//1X,*MEAN NOT CONSIDERED*)
  518 FORMAT(//1X,*IT*,10X,*W(I)*,10X,*WW(I)*,10X,*AA(I)*,10X,*BB(I)*,10
     1X,*XX(IT,I)*//)
  6   FORMAT(1X,*ORDER OF ARIMA PROCESS CONSIDERED IS (*,3I2,*)*)
  61  FORMAT(1X,*PI=*,F7.4,*PIMAX=*,F12.4,*F2=*,F6.3,*ETA=*,F8.5,*DEL=*,
     1F6.3)
      FORMAT(1X,*IT*,6X,*USS*,20X,*BETA*//)
      FORMAT(2X,I2,1X,D16.9,7X,8E12.4)
  451 FORMAT(1X,*50 ITERATIONS TERMINATE THE PROGRAM*)
      FORMAT(1X,*MATRIX IS SINGULAR*)
   12 FORMAT(1X,*USS=*,D20.8)
  4   FORMAT(1X,*CHECK AROUND REGION OF MINIMUM SUM OF SQUARES*//1X,*USE
     1PI TE(I)+-ETA(TOLERANCE)*)
  6   FORMAT(1X,*PI GOT TOO BIG*)
   1  FORMAT(1X,*THE SOLUTION CONVERGED AT THE END OF ITERATION*,I5)
  122 FORMAT(2F6.3,6X,F14.3)
  1   FORMAT(1X,12E10.3)
```

```
 636 FORMAT(5E16.7)
  10 FORMAT(1X,*RESIDUAL (WHITE NOISE) VARIANCE=*,F20.8)
 670 FORMAT(1X,*MATRIX V SINGULAR*)
  17 FORMAT(1X,*THE OVERALL CONSTANT TERM =*,F20.8)
 694 FORMAT(//1X,*RESIDUAL VARIANCE=*,E25.8)
 695 FORMAT(//1X,*COVARIANCE MATRIX OF ESTIMATES*)
 696 FORMAT(1X,*CORRELATION MATRIX OF ESTIMATES*)
 702 FORMAT(1X,*RESIDUAL AUTOCORRELATION FUNCTION*)
 704 FORMAT(7X,*LAG*,2X,12I8,/7X,*ACF*,3X,12E9.2)
 705 FORMAT(//1X,*CHI-SQUARED=*,F10.3,*  DOF=*,I6)
 709 FORMAT(1X,*STANDARD ERROR (SQRT(1/N)) =+-*,E20.8)
    1THIS REMAINING IN RESIDUAL SERIES=*,F20.8)
 712 FORMAT(//1X,*VARIANCE OF DIFFERENCED SERIES =*,E20.8,*PERCENT OF
     STOP
     END
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *


*.*.*.*.*.*.*.*.*SUBROUTINE  UNCONS     *.*.*.*.*.*.*.*.*.*.*
  THIS SUBROUTINE CALCULATES RESIDUAL SERIES AND RESIDUAL SUM OF SQUARES FOR
   A GENERAL ARIMA MODEL OF ORDER  (P,D,Q)


* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      SUBROUTINE UNCONS (NP,NQ,NTERMS,NN,NDIF,KK,ORIGIN,KOUNT,SM,PHI,THE
     1TA,X,DEL,BETA,AA,W,MEAN,USS)
      DOUBLE PRECISION USS
      INTEGER ORIGIN
      DIMENSION XX(3,375),AA(375),EE(375)
      DIMENSION W(375),WW(375),PHI(4),THETA(4),BETA(4)
      REAL WW
      IF(KK.NE.2) NPARAM=1
      IF(KK.EQ.2) NPARAM=NP+NQ+MEAN
      DO 404 NT=1,NPARAM
      N=NTERMS
      IF(KK.EQ.1) GO TO 414
      IF(KK.EQ.1) GO TO 416
      BETA(NT)=BETA(NT)+DEL
 414  CONTINUE
      IF(NP.LT.1) GO TO 417
      DO 418 I=1,NP
 418  PHI(I)=BETA(I)
 417  IF(NQ.LT.1) GO TO 840
      DO 419 I=1,NQ
      INNP=I+NP
 419  THETA(I)=BETA(INNP)
 404  CONTINUE
 840  CONTINUE
C     ADJUST ORIGIN AND LENGTH SERIES
      ORIGIN=NQ
```

```
      IF(NP.GT.0) ORIGIN=NP+NQ+20
      NB=ORIGIN+1
      NN=N+ORIGIN
      DO 406 I=NB,NN
      IFF=I-ORIGIN
      WW(I)=W(IFF)-SM
  406 CONTINUE
      DO 407 I=1,375
  407 EE(I)=0.0

C     CALCULATION OF UNCONDIYIONAL SUM OF SQUARES.  BACKFORECASTING

      JE=NN-NP
      DO 408 J=1,JE
      I=NN-NP-J+1
      SUMMA=0.0
      SUMAR=0.0
      IF(NP.EQ.0) GO TO 409
      DO 410 II=1,NP
      IQII=I+II
  410 SUMAR=SUMAR+PHI(II)*WW(IQII)
  409 IF(NQ.EQ.0) GO TO 412
      DO 411 JJ=1,NQ
      IFJJ=I+JJ
  411 SUMMA=SUMMA+THETA(JJ)*EE(IFJJ)
  412 IF(I.GT.ORIGIN) GO TO 415
      WW(I)=EE(I)+SUMAR-SUMMA
      GO TO 408
  415 CONTINUE
      EE(I)=WW(I)-SUMAR+SUMMA
  408 CONTINUE

C     FORWARD RECURSION TO OBTAIN AA(I) SERIES.

      IF(KK.EQ.2) GO TO 402
      AA(1)=WW(1)
      USS=AA(1)**2
  402 CONTINUE
      IF(KK.EQ.2) XX(NT,1)=WW(1)
      DO 420 I=2,NN
      SUMAR=0.0
      SUMMA=0.0
      IF(NP.EQ.0) GO TO 422
      NF=NP
      IF(NF.GT.I-1) NF=I-1
      DO 424 J=1,NF
      IJJ=I-J
  424 SUMAR=SUMAR+WW(IJJ)*PHI(J)
  422 IF(NQ.EQ.0) GO TO 426
      NI=NQ
```

```
      IF(NE.GT.I-1) NE=I-1
      DO 425 J=1,NE
      IQJ=I-J
      IF(KK.NE.2) TEMP=AA(IQJ)
      IF(KK.EQ.2) TEMP=YX(NT,IQJ)
  425 SUMMA=SUMMA+TEMP*THETA(J)
  426 CONTINUE
      TEMP=WW(I)-SUMAR+SUMMA
      IF(KK.EQ.2) GO TO 420
      USS=USS+TEMP**2
      AA(I)=TEMP
      GO TO 427
  420 YX(NT,I)=TEMP
  427 CONTINUE
      IF(KK.NE.2) GO TO 861
      BETA(NT)=BETA(NT)-DEL
      IF(NP.EQ.0) GO TO 431
      DO 432 I=1,NP
  432 PHI(I)=BETA(I)
  431 IF(NQ.EQ.0) GO TO 861
      DO 434 I=1,NQ
      IPNP=I+NP
  434 THETA(I)=BETA(IPNP)
  861 CONTINUE
      IF(KOUNT.GT.0) GO TO 850
      IF(NP.GT.0) PRINT 856,(PHI(I),I=1,NP)
      IF(NQ.GT.0) PRINT 857,(THETA(I),I=1,NQ)
  856 FORMAT(1X,*AR PARAMETERS PHI(I)=*,4E20.8)
  857 FORMAT(1X,*MA PARAMETERS THETA(I)=*,4E20.8)
  850 CONTINUE
      KOUNT=KOUNT+1
  4   CONTINUE
      RETURN
      END

C --------------------------------------------------------------------------------
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

```
***   PROGRAM NO. 5      .----MULTIVARIATE TIME SERIES ANALYSIS.  ------
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*


**********       MULTIVARIATE TIME SERIES ANALYSIS       **********
*******       THE METHOD IS DESCRIBED IN CHAPTER 4.  THIS IS A MODIFICATIONP
          OF THE METHOD PROPOSED EARLIER BY PHADKE ET AL.

*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
   THIS PROGRAM CALCULATES ORTHOGONAL VECTORC BY GRAM-SCHMIDR ORTHOGONALIZATI
    PROCEDURE.   SINCE THE ORTHOGONAL VECTORS WERE FOUND TO BE SERIALLY AND
    MUTUALLY INDEPENDENT THE PROGRAM USES THEM DIRECTLY IN MULTIPLE REGRESSIC
     ANALYSIS.  SUBROUTINE REGRES IS USED FOR REGRESSION ANALYSIS.  THE METHC
     THE METHOD OF CENTRALIZATION AND CORRELATION MATRIX IS THEREBU USED.
     THE CROSS-CORRELATION FUNCTION BETWEEN THE ORTHOGONAL VECTORS IS
     CALCULATED BY THE SUBROUTINE CCRELA.
* * * * ** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *


*-*-*-  NOTATIONS  *-*-*-
*-*-   BETA(L,I)   UNIVARIATE RESIDUALS.
*-*-  GAMMA(L,I)    ORTHOGONAL VECTOTS.
*/*/    ALPHA(L,I)    ORTHONORMAL VECTORS.
*-*-    SM(LL)   MEAN OF ORIGINAL SERIES.
*-*-    SV(LL)   VARIANCE OF THE ORIGINAL SERIES.
*-*-    SGM(LL)   MEAN OF THE ORTHOGONAL VECTORS.
*-*-   SGV(LL)    VARIANCE OF HTE ORTHOGONAL SERIES.
 *-*     DTPRI(L,J)   DOT PRODUCT OF ORTHOGONAL VECTORS L AND J .


* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION SM(6),SV(6),SGM(6),SGV(6)
      DIMENSION DTPR1(6,6)
      DIMENSION BETA(6,350),GAMMA(6,350),ALPHA(6,350)
      DIMENSION YY(6,350),S(6,350)
      DIMENSION YYY(350),XX(350,10),B(10)
      COMMON YYY,XX
      READ 101,NSAMP,NVAR
  101 FORMAT(20I5)
      DO 809 LM=1,NVAR
      IF (LM.LE.2) READ 102,(BETA(LM,I),I=1,NSAMP)
      IF (LM.EQ.3) GO TO 1111
      IF (LM.GE.4) READ 102,(BETA(LM,I),I=1,NSAMP)
      GO TO 809
 1111 READ 102,(BETA(LM,I),I=1,40)
      READ 102,(BETA(LM,I),I=41,43)
      READ 102,(BETA(LM,I),I=44,NSAMP)
  102 FORMAT(5F16.7)
      SUM1=0.0
      DO 812 I=1,43
```

```
      SUM1=SUM1+BETA(3,I)
  812 CONTINUE
      SUM1=SUM1/43.0
      SVV1=0.0
      DO 813 I=1,43
      SVV1=SVV1+(BETA(3,I)-SUM1)**2
  813 CONTINUE
      SVV1=SVV1/43.0
      DO 814 I=1,43
      BETA(3,I)=(BETA(3,I)-SUM1)/SQRT(SVV1)
  814 CONTINUE
      SUM2=0.0
      DO 815 I=44,223
      SUM2=SUM2+BETA(3,I)
  815 CONTINUE
      SUM2=SUM2/180.0
      SVV2=0.0
      DO 816 I=44,223
      SVV2=SVV2+(BETA(3,I)-SUM2)**2
  816 CONTINUE
      SVV2=SVV2/180.0
      DO 817 I=44,223
      BETA(3,I)=(BETA(3,I)-SUM2)/SQRT(SVV2)
  817 CONTINUE
      SUM3=0.
      DO 818 I=224,350
      SUM3=SUM3+BETA(3,I)
  818 CONTINUE
      SUM3=SUM3/127.0
      SVV3=0.
      DO 819 I=224,350
      SVV3=SVV3+(BETA(3,I)-SUM3)**2
  819 CONTINUE
      SVV3=SVV3/127.0
      DO 820 I=224,350
      BETA(3,I)=(BETA(3,I)-SUM3)/SQRT(SVV3)
  820 CONTINUE
      PRINT 821
  821 FORMAT(1X,* THE MEAN AND VARIANCE OF THE THREE PARTS OF THE BLAST
     , 1FLOW RATE SERIES*)
      PRINT 100,SUM1,SUM2,SUM3,SVV1,SVV2,SVV3
  100 FORMAT(6F16.7)
  809 CONTINUE
      DO 301 I=1,NSAMP
      GAMMA(1,I)=BETA(1,I)
  301 CONTINUE
      SUM1=0.
      DO 302 I=1,NSAMP
      SUM1=SUM1+GAMMA(1,I)**2
  302 CONTINUE
```

```
      SMODE=SQRT(SUM1)
      DO [illegible] I=1,NSAMP
      ALPHA(1,I)=GAMMA(1,I)/SMODE
[illegible] CONTINUE
      DO [illegible] I=1,NSAMP
      DO [illegible] L=2,NVAR
      S(L,I)=0.0
[illegible] CONTINUE
      DO [illegible] J=2,NVAR
      JJ=J-1
      DO [illegible] K=1,JJ
      KK=K+1
      SUM=0.
      DO [illegible] I=1,NSAMP
      SUM=SUM+ALPHA(K,I)*BETA(J,I)
[illegible] CONTINUE
      [illegible] SUM
[illegible] FORMAT(2X,*SUM =*,E16.7)
      DO 12[illegible] I=1,NSAMP
      YY(KK,I)=SUM*ALPHA(K,I)
12[illegible] CONTINUE
15[illegible] CONTINUE
      DO [illegible] I=1,NSAMP
      DO [illegible] L=2,J
      S(J,I)=S(J,I)+YY(L,I)
2[illegible] CONTINUE
      DO [illegible] I=1,NSAMP
      DO [illegible] M=2,J
      GAMMA(M,I)=BETA(M,I)-S(M,I)
5[illegible] CONTINUE
      SS=0.0
      DO [illegible] I=1,NSAMP
      SS=SS+GAMMA(J,I)**2
7[illegible] CONTINUE
      DO [illegible] I=1,NSAMP
      ALPHA(J,I)=GAMMA(J,I)/SQRT(SS)
7[illegible] CONTINUE
6[illegible] CONTINUE
      DO [illegible] LL=1,NVAR
      SM(LL)=0.0
      DO [illegible]1 I=1,NSAMP
      SM(LL)=SM(LL)+BETA(LL,I)
[illegible] CONTINUE
      SM(LL)=SM(LL)/FLOAT(NSAMP)
8[illegible]2 CONTINUE
      DO [illegible]4 LL=1,NVAR
      SV(LL)=0.0
      DO [illegible] I=1,NSAMP
      SV(LL)=SV(LL)+(BETA(LL,I)-SM(LL))**2
80[illegible] CONTINUE
```

```
      SV(LL)=SV(LL)/FLOAT(NSAMP)
[illegible] CONTINU
      [illegible] LL=1,NVAR
      SGM(LL)=[illegible].
      DO [illegible] I=1,NSAMP
      SGM(LL)=SGM(LL)+GAMMA(LL,I)
[illegible] CONTINU
      SGM(LL)=SGM(LL)/FLOAT(NSAMP)
8[illegible] CONTINUE
      DO 808 LL=1,NVAR
      SGV(LL)=[illegible].0
      DO 807 I=1,NSAMP
      SGV(LL)=SGV(LL)+(GAMMA(LL,I)-SGM(LL))**2
807   CONTINUE
      SGV(LL)=SGV(LL)/FLOAT(NSAMP)
8[illegible]  CONTINU
      PRINT 810
      PRINT 360,(SM(LL),LL=1,NVAR)
      PRINT 360,(SV(LL),LL=1,NVAR)
      PRINT 811
      PRINT 360,(SGM(LL),LL=1,NVAR)
      PRINT 360,(SGV(LL),LL=1,NVAR)
810   FORMAT(1X,*THE MEAN AND VARIANCE OF ORIGINAL VECTORS*)
811   FORMAT(1X,*THE MEAN AND VARIANCE OF ORTHOGONAL VECTORS*)
      DO 160 L=1,NVAR
      DO 160 J=1,NVAR
      DTPR1(L,J)=0.0
      DO 160 I=1,NSAMP
      DTPR1(L,J)=DTPR1(L,J)+GAMMA(L,I)*GAMMA(J,I)
16[illegible] CONTINU
      DO 827 L=1,NVAR
      PRINT 360,(DTPR1(L,J),J=1,NVAR)
827   CONTINUE
      DO [illegible] I=1,NSAMP
      PRINT [illegible],(GAMMA(L,I),L=1,NVAR)
65[illegible] CONTINUE
360   FORMAT(1X,7F15.7)
      DO 6[illegible] I=1,NSAMP
      YYY(I)=BETA(2,I)-GAMMA(2,I)
      [illegible](I,1)=GAMMA(1,I)
6[illegible] CONTINUE
      NORD=1
      CALL [illegible]GR[illegible]S (NSAMP,NORD,B)
      DO [illegible] I=1,NSAMP
      GAMMA(2,I)=BETA(2,I)-B(1)*GAMMA(1,I)
6[illegible]2 CONTINUE
      PRINT [illegible],(GAMMA(2,I),I=1,NSAMP)
37[illegible] FORMAT(5F16.7)
      PUNCH [illegible],(GAMMA(2,I),I=1,NSAMP)
      DO 6[illegible] I=1,NSAMP
```

```
      YYY(I)=BETA(2,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
  604 CONTINUE
      NORD=2
      CALL REGRES (NSAMP,NORD,B)
      DO 605 I=1,NSAMP
      YYY(I)=BETA(3,I)-GAMMA(3,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
  605 CONTINUE
      NORD=2
      CALL REGRES (NSAMP,NORD,B)
      DO 606 I=1,NSAMP
      GAMMA(3,I)=BETA(3,I)-B(1)*GAMMA(1,I)-B(2)*GAMMA(2,I)
  606 CONTINUE
      PRINT 360,(GAMMA(3,I),I=1,NSAMP)
      PUNCH 370,(GAMMA(3,I),I=1,NSAMP)
      DO 608 I=1,NSAMP
      YYY(I)=BETA(3,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
  608 CONTINUE
      NORD=3
      CALL REGRES (NSAMP,NORD,B)
      DO 609 I=1,NSAMP
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      YYY(I)=BETA(4,I)-GAMMA(4,I)
  609 CONTINUE
      NORD=3
      CALL REGRES (NSAMP,NORD,B)
      DO 610 I=1,NSAMP
      GAMMA(4,I)=BETA(4,I)-B(1)*GAMMA(1,I)-B(2)*GAMMA(2,I)-B(3)*GAMMA(3,
     1I)
  610 CONTINUE
      PRINT 360,(GAMMA(4,I),I=1,NSAMP)
      PUNCH 370,(GAMMA(4,I),I=1,NSAMP)
      DO 611 I=1,NSAMP
      YYY(I)=BETA(4,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      XX(I,4)=GAMMA(4,I)
  611 CONTINUE
      NORD=4
      CALL REGRES (NSAMP,NORD,B)
      DO 612 I=1,NSAMP
```

```
      YYY(I)=BETA(5,I)-GAMMA(5,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      XX(I,4)=GAMMA(4,I)
  613 CONTINUE
      NORD=4
      CALL REGRES(NSAMP,NORD,B)
      DO 614 I=1,NSAMP
      GAMMA(5,I)=BETA(5,I)-B(1)*GAMMA(1,I)-B(2)*GAMMA(2,I)-B(3)*GAMMA(3,
     2I)-B(4)*GAMMA(4,I)
  614 CONTINUE
      PRINT 360,(GAMMA(5,I),I=1,NSAMP)
      PUNCH 370,(GAMMA(5,I),I=1,NSAMP)
      DO 616 I=1,NSAMP
      YYY(I)=BETA(5,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      XX(I,4)=GAMMA(4,I)
      XX(I,5)=GAMMA(5,I)
  616 CONTINUE
      NORD=5
      CALL REGRES(NSAMP,NORD,B)
      DO 617 I=1,NSAMP
      YYY(I)=BETA(6,I)-GAMMA(6,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      XX(I,4)=GAMMA(4,I)
      XX(I,5)=GAMMA(5,I)
  617 CONTINUE
      NORD=5
      CALL REGRES (NSAMP,NORD,B)
      DO 618 I=1,NSAMP
      GAMMA(6,I)=BETA(6,I)-B(1)*GAMMA(1,I)-B(2)*GAMMA(2,I)-B(3)*GAMMA(3,
     3I)-B(4)*GAMMA(4,I)-B(5)*GAMMA(5,I)
  618 CONTINUE
      PRINT 360,(GAMMA(6,I),I=1,NSAMP)
      PUNCH 370,(GAMMA(6,I),I=1,NSAMP)
      DO 620 I=1,NSAMP
      YYY(I)=BETA(6,I)
      XX(I,1)=GAMMA(1,I)
      XX(I,2)=GAMMA(2,I)
      XX(I,3)=GAMMA(3,I)
      XX(I,4)=GAMMA(4,I)
      XX(I,5)=GAMMA(5,I)
      XX(I,6)=GAMMA(6,I)
  620 CONTINUE
      NORD=6
```

```
      CALL REGRES (NSAMP,NORD,B)
      STOP
      END

*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
C     SUBROUTINE REGRES
      THIS SUBROUTINE IS CALLED BY THE MAIN PROGRAM FOR REGRESSION ANALYSIS.
        NSAMP    NUMBER OF THE DATA POINTS.
        NORD     THE NO. OF INDEPENDENT VARIABLES.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*

      SUBROUTINE REGRES (NSAMP,NORD,BETA)
      DIMENSION Y(350),X(350,10),XTRP(10,350),YEST(350),XTRPX(10,10)
      DIMENSION XMEAN(10),XTRPY(10),BETA(10),B(10),BB(10,1),SUMX(10),
     1S(10)
      COMMON Y,X
      REAL MESQR
      IT=0
      DO 30 L=1,NORD
      SUMX(L)=0.0
      DO 20 I=1,NSAMP
      SUMX(L)=SUMX(L)+X(I,L)
   20 CONTINUE
      XMEAN(L)=SUMX(L)/FLOAT(NSAMP)
   30 CONTINUE
      SUMY=0.0
      DO 40 I=1,NSAMP
      SUMY=SUMY+Y(I)
   40 CONTINUE
      YMEAN=SUMY/FLOAT(NSAMP)
      PRINT 105,(XMEAN(L),L=1,NORD),YMEAN
      DO 330 L=1,NORD
      DO 330 I=1,NSAMP
      X(I,L)=X(I,L)-XMEAN(L)
  330 CONTINUE
      DO 50 I=1,NSAMP
      Y(I)=Y(I)-YMEAN
   50 CONTINUE
  130 CONTINUE
      DO 60 I=1,NSAMP
      DO 60 L=1,NORD
      XTRP(L,I)=X(I,L)
   60 CONTINUE
      DO 70 I=1,NORD
      DO 70 L=1,NORD
      XTRPX(I,L)=0.0
      DO 70 K=1,NSAMP
      XTRPX(I,L)=XTRPX(I,L)+XTRP(I,K)*X(K,L)
```

```
 70 CONTINUE
    IF (IT.EQ.1) GO TO 110
    IT=1
    SYY=0.0
    DO 80 I=1,NSAMP
    SYY=SYY+Y(I)**2
 80 CONTINUE
    DO 90 L=1,NORD
    S(L)=XTRPX(L,L)
 90 CONTINUE
    PRINT 105,SYY,(S(L),L=1,NORD)
    DO 100 I=1,NSAMP
    Y(I)=Y(I)/SQRT(SYY)
100 CONTINUE
    DO 120 L=1,NORD
    DO 120 I=1,NSAMP
    X(I,L)=X(I,L)/SQRT(S(L))
120 CONTINUE
    GO TO 130
110 CALL MATINV (XTRPX,NORD,BB,0,DETRM)
    DO 140 L=1,NORD
    XTRPY(L)=0.0
140 CONTINUE
    DO 150 I=1,NORD
    DO 150 J=1,NSAMP
    XTRPY(I)=XTRPY(I)+XTRP(I,J)*Y(J)
150 CONTINUE
    DO 160 L=1,NORD
    B(L)=0.0
160 CONTINUE
    DO 170 L=1,NORD
    DO 170 J=1,NORD
    B(L)=B(L)+XTRPX(L,J)*XTRPY(J)
170 CONTINUE
    PRINT 106
    PRINT 105,(B(I),I=1,NORD)
    DO 180 L=1,NORD
    BETA(L)=B(L)*SQRT(SYY/S(L))
180 CONTINUE
    PRINT 107
    PRINT 105,(BETA(L),L=1,NORD)
    SUMYY=0.0
    DO 310 I=1,NSAMP
    SUMYY=SUMYY+Y(I)
310 CONTINUE
    YMNOR=SUMYY/FLOAT(NSAMP)
    PRINT 250,YMNOR
    SSREG=0.0
    DO 190 I=1,NSAMP
    YEST(I)=0.0
```

```
      DO 200 J=1,NORD
      YEST(I)=YEST(I)+B(J)*X(I,J)
  200 CONTINUE
      SSREG=SSREG+(YEST(I)-YMNOR)**2
  190 CONTINUE
      SSMEAN=0.0
      DO 320 I=1,NSAMP
      SSMEAN=SSMEAN+(Y(I)-YMNOR)**2
  320 CONTINUE
      RESIDU=SSMEAN-SSREG
      NDFTOT=NSAMP-1
      NDFREG=NORD
      NDFRES=NDFTOT-NDFREG
      MESQR=SSREG/FLOAT(NDFREG)
      VARIAN=RESIDU/FLOAT(NDFRES)
      FVALUE=MESQR/VARIAN
      PRINT 108
      PRINT 104
      PRINT 101,SSMEAN,NDFTOT
      PRINT 102,SSREG,NDFREG,MESQR,FVALUE
      PRINT 103,RESIDU,NDFRES,VARIAN
  101 FORMAT(2X,*TOTAL(CORRECTED FOR MEAN)*,2X,E16.7,2X,I5//)
  102 FORMAT(12X,*REGRESSION*,5X,E16.8,10X,I5,5X,E16.8,5X,E16.8,5X,E16.8
     1//)
  103 FORMAT(13X,*RESIDUAL*,5X,E16.8,10X,I5,5X,E16.8)
  104 FORMAT(15X,*SOURCE*,8X,*SUM OF SQUARES*,5X,*DEGREES OF FREEDOM*,5X
     1,*MEAN SQUARE*,5X,*F VALUE*//)
  105 FORMAT(1X,10E13.5)
  106 FORMAT(1X,*THE OLD PARAMETERS ARE*)
  107 FORMAT(1X,*THE ACTUAL PARAMETERS OF THE MODEL ARE*)
  108 FORMAT(50X,*ANALYSIS  OF  VARIANCE  TABLE*)
  109 FORMAT(1X,*THE X TRANSPOSE X MATRIX*)
  250 FORMAT(1X,*MEAN OF NORMALIZED Y(I) SERIES  =*,E16.8)
      RETURN
      END
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*

C    SUBROUTINE  CORELA .
     THIS SUBROUTINE CALCULATES CROSS-CORRELATION FUNCTION BETWEEN ALL PAIRS
     THE VECTORS.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
     MLAG ... MAXIMUM NO. OF LAGS.
     RP(I) ...  CORRELATION COEFFICIENT.
     WP(I) ... COVARIANCE COEFFICIENT.
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
      SUBROUTINE CORELA (NSAMP,MLAG,NVCTR,X,RP,WP)
      DIMENSION CP(40),WP(40),RP(40)
      DIMENSION X(12,350),SP(350),GP(350),TP(350),FP(350)
```

```
      DO 300 IX=1,NVCTR
      DO 300 IY=1,NVCTR
      TP(1)=0.0
      GP(1)=0.0
      FP(1)=0.0
      SP(1)=0.0
      DO 16 I=1,NSAMP
      SP(1)=SP(1)+X(IX,I)**2
      TP(1)=TP(1)+X(IX,I)
      GP(1)=GP(1)+X(IY,I)**2
      FP(1)=FP(1)+X(IY,I)
   16 CONTINUE
      M1=MLAG+1
      DO 20 I=2,M1
      J=I-1
      K=NSAMP-I+2
      TP(I)=TP(J)-X(IX,J)
      SP(I)=SP(I-1)-X(IX,J)**2
      FP(I)=FP(J)-X(IY,K)
      GP(I)=GP(I-1)-X(IY,K)**2
   20 CONTINUE
      MLAG1=MLAG+1
      DO 30 I=1,MLAG1
      NMINP=NSAMP-I+1
      CP(I)=0.0
      DO 26 J=1,NMINP
      K2=J+I-1
   26 CP(I)=CP(I)+(X(IX,K2)*X(IY,J))
      WP(I)=CP(I)/FLOAT(NMINP)
      RNUM=FLOAT(NMINP)*CP(I)-FP(I)*TP(I)
      RDEN1=SQRT((FLOAT(NMINP)*GP(I))-FP(I)**2)
      RDEN2=SQRT((FLOAT(NMINP)*SP(I))-TP(I)**2)
   30 RP(I)=RNUM/(RDEN1*RDEN2)
      IF(IX-IY) 202,203,202
  202 PRINT 150,IX,IY
      GO TO 204
  203 PRINT 99,IX,IY
  204 CONTINUE
   99 FORMAT(1X,*AUTO CORRELATION FUNCTION BETWEEN X=*,I4,*AND Y=*,I4)
  150 FORMAT(1X,*CROSS CORRELATION FUNCTION BETWEEN X=*,I4,*AND Y=*,I4)
      PRINT 660,(RP(I),I=1,MLAG)
  660 FORMAT(1X,16F8.4)
      CHISQ1=0.0
      CHISQ2=0.0
      CHISQ3=0.0
      DO 60 ICHI=1,30
      II=ICHI+1
      IF (II.LE.13) CHISQ1=CHISQ1+RP(II,IX,IY)**2
      IF (II.LE.25) CHISQ2=CHISQ2+RP(II,IX,IY)**2
      IF (II.LE.31) CHISQ3=CHISQ3+RP(II,IX,IY)**2
```

```
 61  CONTINUE
     TCHI1=CHISQ1*FLOAT(NSAMP)
     TCHI2=CHISQ2*FLOAT(NSAMP)
     TCHI3=CHISQ3*FLOAT(NSAMP)
 688 FORMAT(1X,5E20.8)
     PRINT 66,TCHI1,TCHI2,TCHI3
  66 FORMAT(1X,*CHI-SQUARE STATISTICS FROM LAG 1 TO 12=*,E20.8,/1X,*CHI
    1-SQUARE STATISTICS FROM LAG 1 TO 24=*,E20.8,/1X,*CHI-SQUARE STATIS
    2TICS FROM LAG 1 TO 36=*,E20.8)
 20  CONTINUE
     RETURN
     END
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-
```